The "ΦTOP": A Golden Ellipsoid

Kenneth Brecher
Departments of Astronomy and Physics
Boston University
Boston, MA 02215, U.S.A.
brecher@bu.edu

Overview

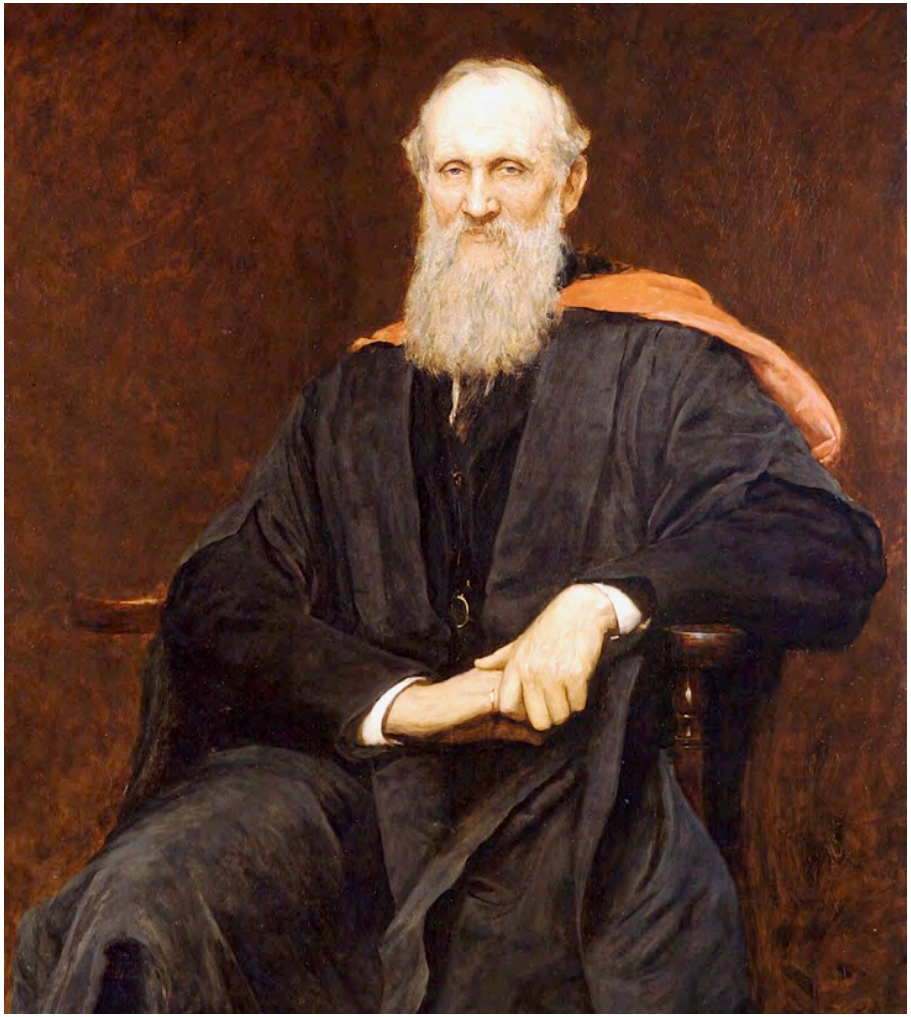
Background of the Problem of the Rise of the Center of Mass of Spinning Objects

Development of the "PhiTOP":
A Golden Ellipsoid

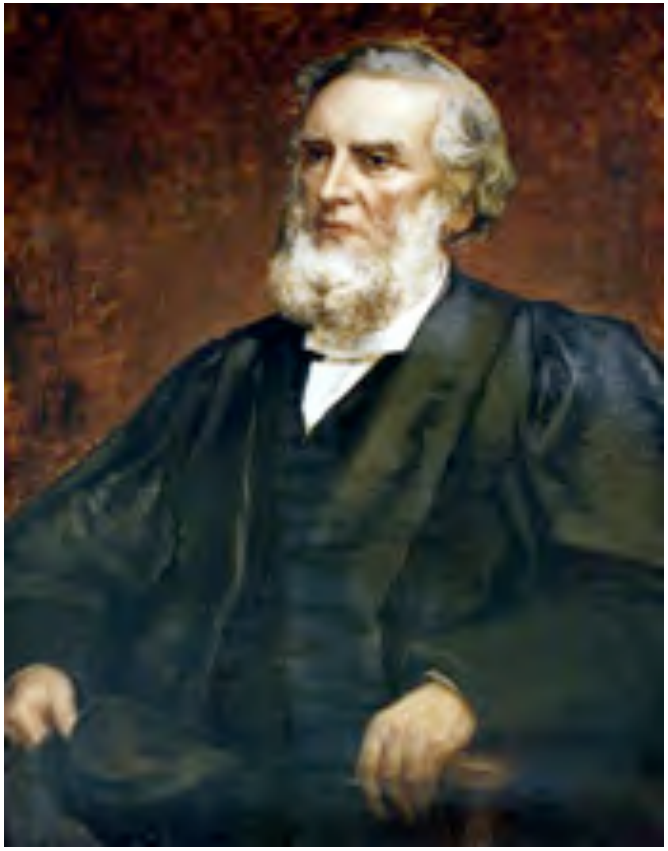
Brief Remarks about The Golden Mean

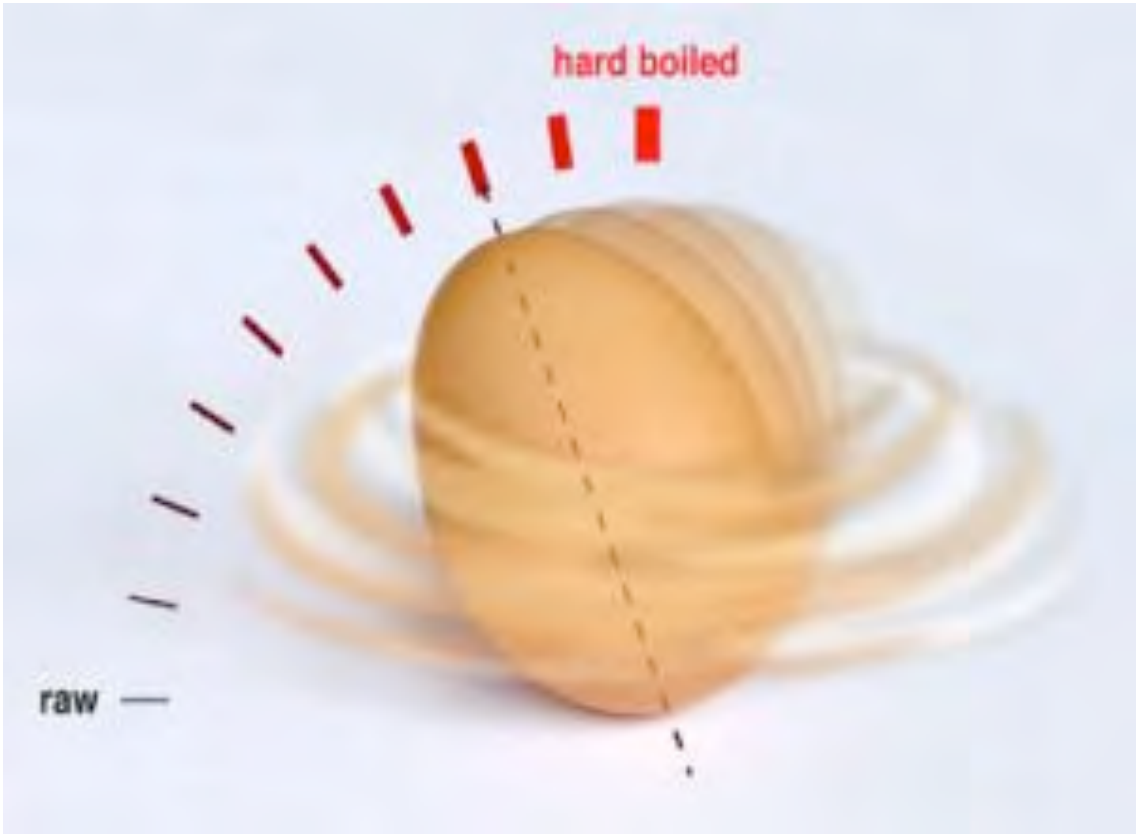
Some Properties of the PhiTOP:
Dynamic, Magnetic, Artistic, Visual, Aesthetic

Lord Kelvin's Pebbles



(John H.) Jellett's Eggs





How can you tell a raw egg from a hard-boiled egg? Hint: spin it!

("Real" Lord) Shiva's Lingam Stones

Lingam stones from the Narmada River in western India

Questions:

What do these objects have in common?

When did the study of the rise of the center of mass (COM) of spinning objects acting under friction and gravity actually begin?

Who has actively worked on this topic?

Has there been any real progress in the analytic treatment and human understanding of such "non-holonomic" systems?

Tippe Top Patent 1891

KAISERLICHES PATENTAMT

AUSGEGEBEN DEN 12. JULI 1892.

PATENTSCHRIFT

— № 63261 —

KLASSE 77: Sport.

FRÄULEIN HELENE SPERL IN MÜNCHEN.

Wendekreisel.

Patentirt im Deutschen Reiche vom 7. October 1891 ab.

Der Erfindungsgegenstand besteht in einem Kreisel, welcher massiv oder hohl, aus Metall, Holz, Papiermaché oder aus einem beliebigen Stoff hergestellt sein kann, dessen untere Fläche beiläufig die Form einer Halbkugel zeigt, während die obere Fläche beliebige Formen (eben, ausgehöhlt oder gewölbt) haben kann. Senkrecht zur Mitte der oberen Fläche und mit dieser fest zusammenhängend ist ein Stiel angeordnet, durch welchen der Kreisel mittelst der Hand in Drehung versetzt wird.

Fig. 1 zeigt schaubildlich einen Kreisel mit ebener Fläche nebst Stiel.

Fig. 2 zeigt schaubildlich einen Kreisel mit hohler Fläche nebst Stiel.

Fig. 3 ist der Durchschnitt in Linie *a-b* der Fig. 1.

Fig. 4 ist der Durchschnitt der Linie *a-b* der Fig. 2.

Fig. 5 zeigt einen Durchschnitt des Kreisels mit gewölbter Oberfläche.

Fig. 6 zeigt einen Durchschnitt des Kreisels mit einer anderen Form der Oberfläche.

Fig. 7 zeigt den Kreisel nach Fig. 1 und 3 mit einer Schraube *K* am Stiel zur Verlegung des Schwerpunktes.

Versetzt man einen solchen Kreisel, wie er durch die Zeichnung dargestellt ist, und dessen Schwerpunkt genau in den Linien *a-b* der Fig. 1 bis 6 liegt, in Drehung, so wird ihm hierbei immer eine kleine Schwankung mitgetheilt. Ist der Winkel, den infolge dessen die geometrische Achse des Kreisels mit der Drehachse einschliefst, so grofs geworden, dafs der Schwerpunkt, der ja in der Linie *a-b* liegt, weit genug von der Rotationsachse entfernt ist, so gewinnen die Kräfte, welche die Verschiebung dieser Achse veranlassen, rasch die Oberhand, und es tritt die scheinbar plötzliche Wendung der unteren Fläche nach oben ein. Wenn nun der Kreisel z. B. auf der unteren Fläche roth, auf der oberen blau bemalt ist, so wirkt das Spielzeug beim Umspringen des Kreisels überraschend schön durch den Farbenwechsel.

Bei der Fabrikation dieses Gegenstandes kommt es hauptsächlich darauf an, dafs neben entsprechender Form der unteren Fläche des Kreisels die Dimensionen des Hauptkörpers zu denen des Stieles in richtigem, durch Versuche festgesetztem Verhältnifs stehen, also auf das Verhältnifs der Schwerpunktlage des Hauptkörpers zur Schwerpunktlage des Stieles. Da dieses Verhältnifs der Schwerpunktlage sehr genau gewählt sein mufs, so kann das Ende des Stieles mit einem etwa halbrunden Kopf versehen sein, der mittelst eines daran befestigten Stiftes *s* höher oder tiefer gestellt werden kann. Zur bequemen Regulirung kann dieser Stift *s* mit einem Gewinde versehen sein. Wählt man einen blanken Kopf, so wird der Effect des Spielzeuges durch glänzende Ringe noch wesentlich erhöht.

PATENT-ANSPRÜCHE:

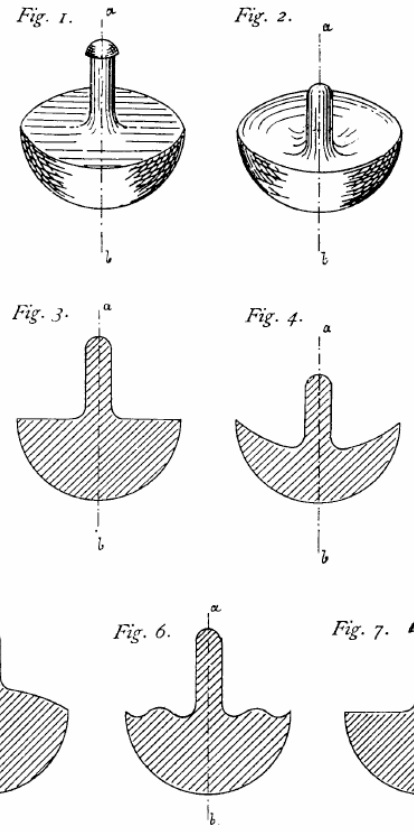
1. Ein hohler oder massiver, unten genau oder nahezu halbkugelförmig gestalteter und oben mit einem Stiel versehener Kreisel, bei dem eine bestimmte Schwerpunktlage bewirkt, dafs er bei Versetzung in Drehung zuerst die mit dem Stiel versehene Seite, sodann aber die halbkugelförmige, anders gefärbte Fläche nach oben kehrt.
2. Für den unter No. 1. gekennzeichneten Kreisel ein oben am Stiel angebrachter, zur Regulirung der Schwerpunktlage dienender, verstellbarer Knopf.

Hierzu 1 Blatt Zeichnungen.

BERLIN. GEDRUCKT IN DER REICHSDRUCKEREI.

FRÄULEIN HELENE SPERL IN MÜNCHEN.

Wendekreisel.



Zu der Patentschrift

№ 63261.

PHOTOGR. DRUCK DER REICHSDRUCKEREI.

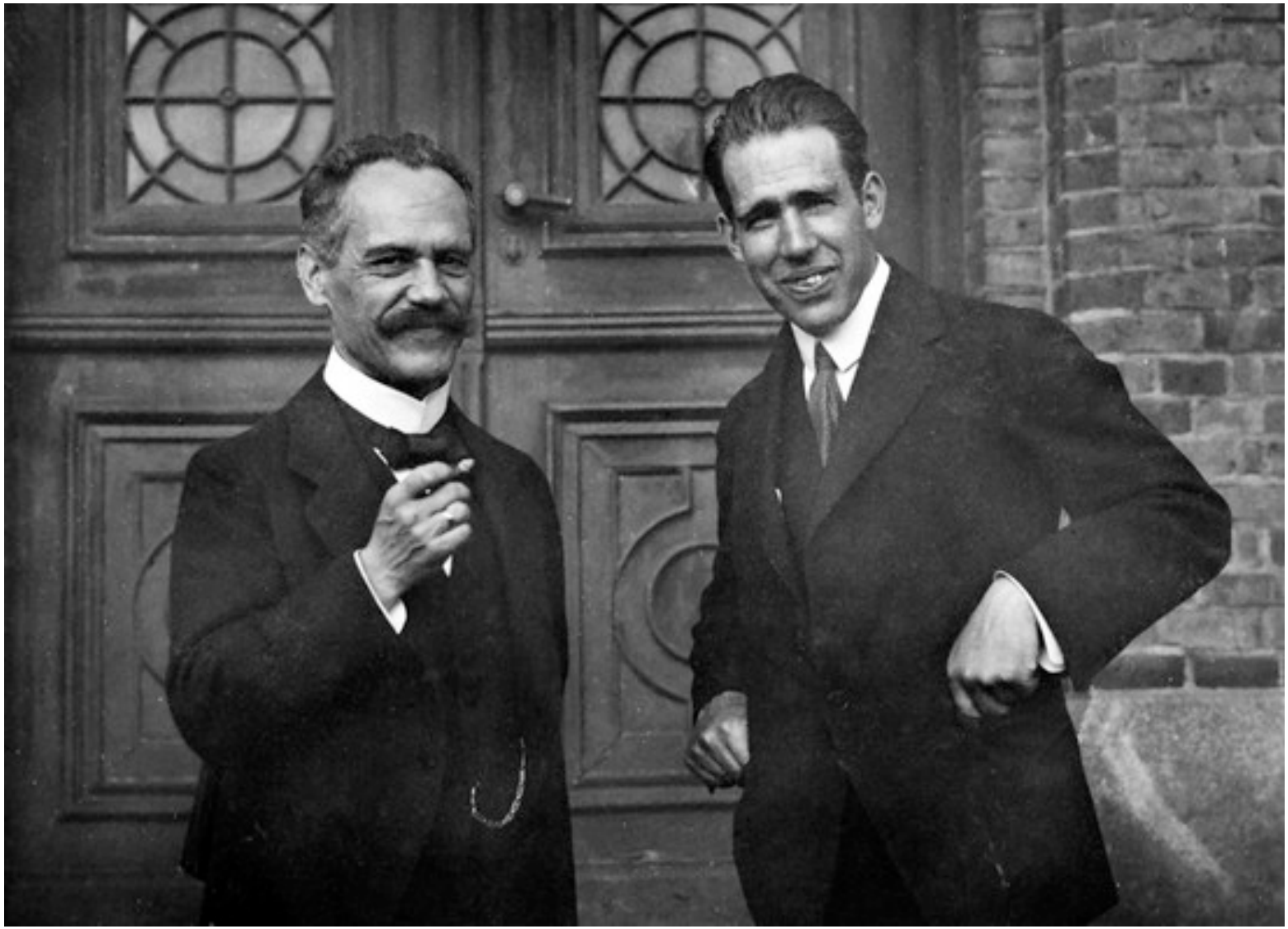
Original patent for "Wenderkreisel", expired 6 months later for non-payment.

Modern Tippe Tops

Felix Klein (l.) and Arnold Sommerfeld (r.) "The Theory of the Top", published between 1896 - 1910, 4 Volumes, ~1000 pages.

They made no attempt to treat problems like the tippe top.

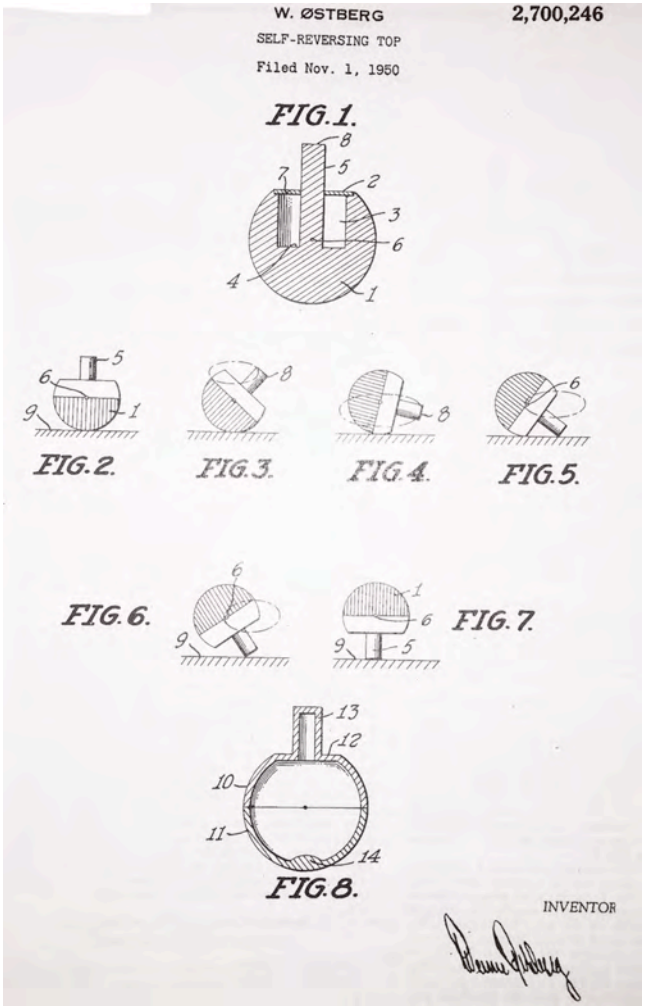
Sommerfeld (l.) and Bohr (r.) in 1919
Reaching for a Tippe Top?

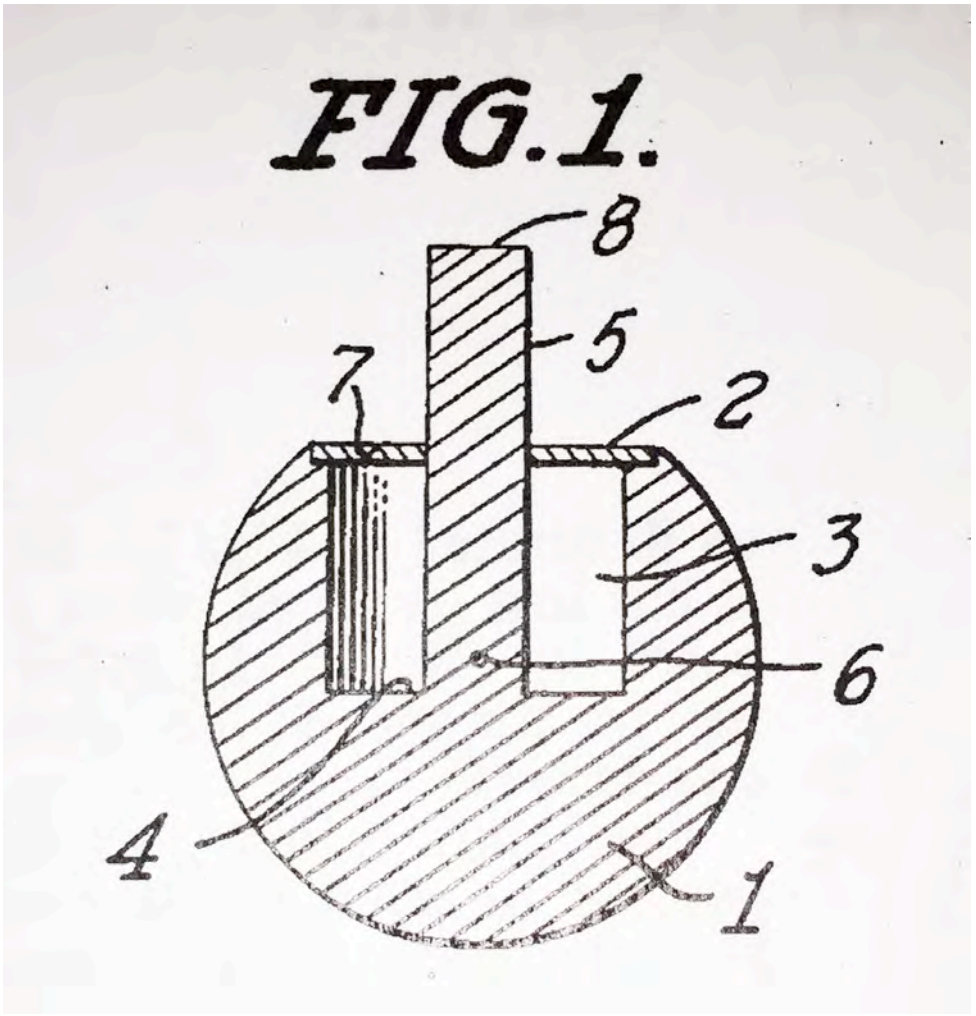


Sommerfeld (l.) and Pauli (r.) in 1934
Reaching for a Tippe Top?

Tippe Top Re-invented ~ 1950

A Danish engineer named Werner Ostberg re-invented and patented it, naming the device the "tippe top" ("tippetoppen" in Danish), 1950


W. ØSTBERG 2,700,246
SELF-REVERSING TOP
Filed Nov. 1, 1950
FIG. 1.
FIG. 2.
FIG. 3.
FIG. 4.
FIG. 5.
FIG. 6.
FIG. 7.
FIG. 8.
INVENTOR



Tippe Tops Sweep the World!

Bohr and Pauli in 1951 (in Lund, Sweden) Playing with a Tippe Top (Yes!)

Bohr Discussing Tippe Top With Swedish King Gustav VI (1951)

Some Questons:

What happened to the tippe top after 1891?

Was it ever made and sold widely?

Was any theory done and published on it?

Did studies of the problem of the rise of the center of mass (COM) for spinning tops start with the invention of the tippe top as often implied in mechanics textbooks?

First Known Discussion of the Rise of the COM of Spinning Top, 1844

William Thompson (later Lord Kelvin), age 22

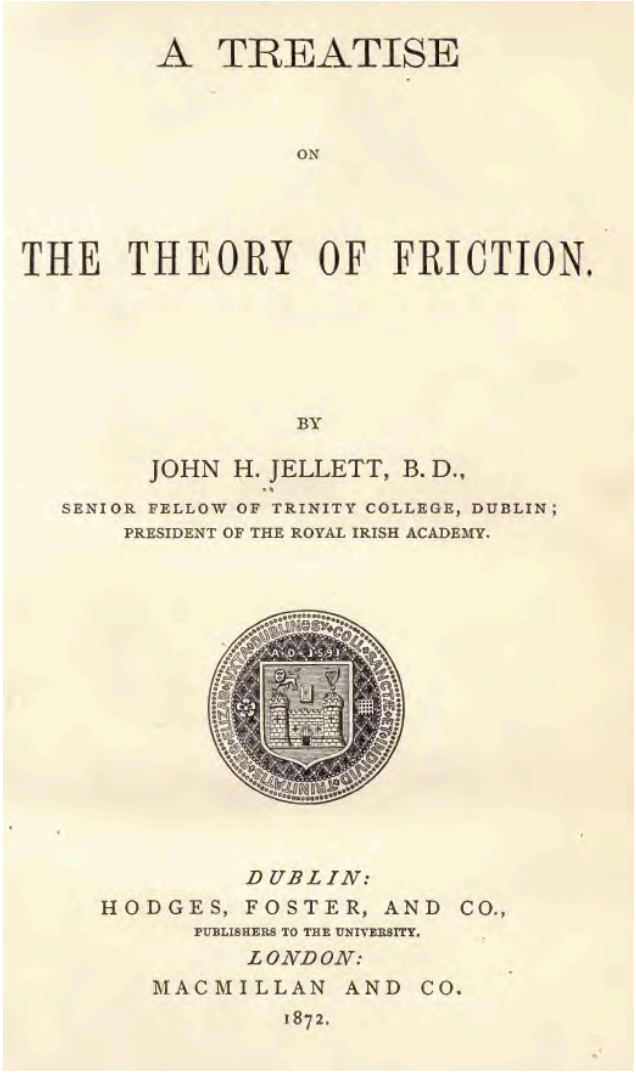
In a letter to his father written while studying for the Tripos exam to be held at Cambridge University in 1844, he wrote that he had spent a great deal of time spinning pebbles and trying to understand the rise of their COM:

"I have been investigating, as far as I have been able in the time I have been able to spare, the theory of spinning tops and rolling hoops, which is very curious and difficult. [Hugh] Blackburn [later Professor of Mathematics in Glasgow] and I have been making a great many experiments on the subject, and have collected a cabinet of ellipsoids of various proportions, which we find on the beach, besides having gotten a teetotum, humming top, and peesy. Some of the results we have obtained are very curious."

In the subsequent Tripos Exam, he came out Second Wrangler because of these activities. He later told his biographer Silvanus P. Thompson:

"I might have made up on the last two days but for my bad generalship. One paper was really a paper that I ought to have walked through, but I did very badly by my bad generalship, and must have got hardly any marks. I spent nearly all the time on one particular problem that interested me, about a spinning top being let fall on to a rigid plane; a very simple problem if I had tackled it in the right way, but I got involved and lost time on it and wrote something that was not good, and there was no time left for the other questions."

J. H. Jellett "Theory of Friction" 1872 Includes a New Adiabatic Invariant



A TREATISE

ON

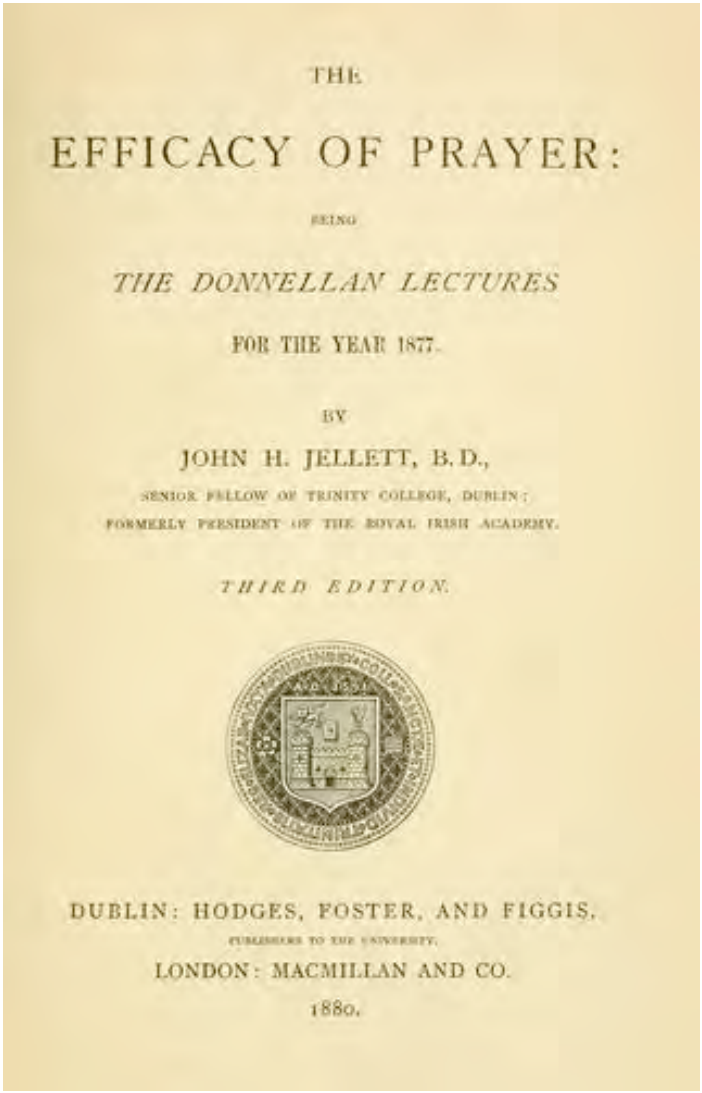
THE THEORY OF FRICTION.

BY

JOHN H. JELLETT, B. D.,

SENIOR FELLOW OF TRINITY COLLEGE, DUBLIN;
PRESIDENT OF THE ROYAL IRISH ACADEMY.

DUBLIN:
HODGES, FOSTER, AND CO.,
PUBLISHERS TO THE UNIVERSITY.
LONDON:
MACMILLAN AND CO.
1872.



THE

EFFICACY OF PRAYER:

BEING

THE DONNELLAN LECTURES

FOR THE YEAR 1877.

BY

JOHN H. JELLETT, B.D.,

SENIOR FELLOW OF TRINITY COLLEGE, DUBLIN;
FORMERLY PRESIDENT OF THE ROYAL IRISH ACADEMY.

THIRD EDITION.

DUBLIN: HODGES, FOSTER, AND FIGGIS,
PUBLISHERS TO THE UNIVERSITY.
LONDON: MACMILLAN AND CO.
1880.

Theory of Tops with Friction Over the Next 140 years

Many theoretical papers written.

No complete analytic solutions found for tippe-tops, spinning eggs (ovoids) or related objects (nor for the related problem of rattlebacks).

Lots of numerical simulations.

Difficulty: no analytic theory of friction: these spinning tops are "non-holonomic" systems.

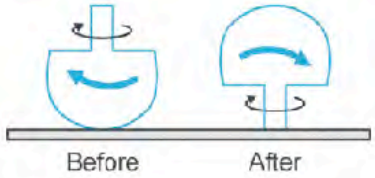
Tippe Top Torture (KB Problem Set)

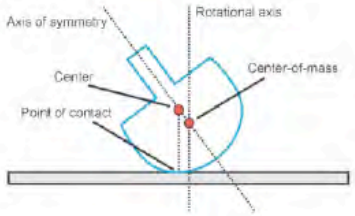
AS 725 Gravitational Astrophysics Spring 2010

Tippy Top Homework Problem

The "Tippy Top" (sometimes also called a Tippe Top") is a remarkable mechanical object that has intrigued physicists (and others) for over a century (cf. photos below featuring Niels Bohr and Wolfgang Pauli).

The top is spun with an initial rotation period T_i. It has mass M and radius R. After a period of time T_f it flips over and spins in the opposite direction, seemingly violating the laws of physics.

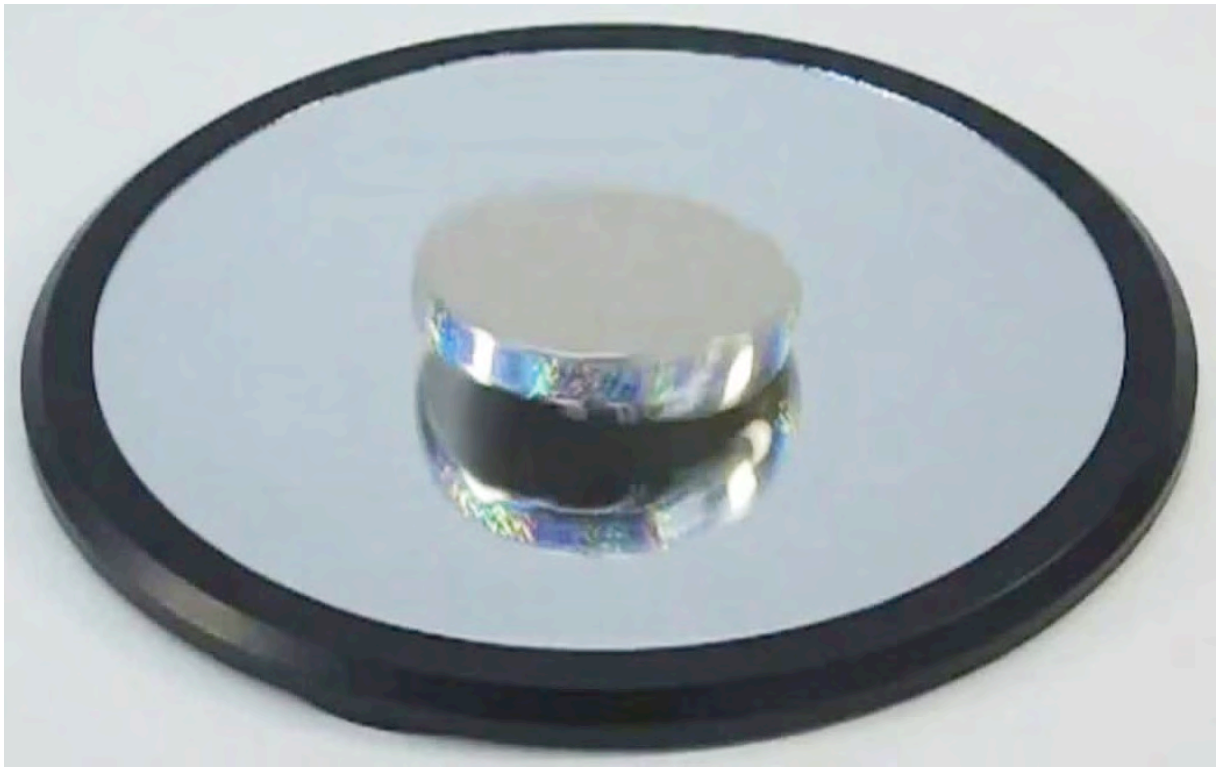




Using the laws of mechanics (F = ma), Newton's law of gravitation $F = Gm_1m_2/d^2$, and a simple model for the frictional contact force $F = \mu mg$, find a rough value for T_f/T_i as a function of the relevant variables. Your answer must include G and the mass of the Earth M_e (after all, this is a course in Gravitational Astrophysics). Using your tippe top, estimate the value of T_i and show that it corresponds to your observed value of T_f.

Extra Credit: Prove that the stable position for the top while it is rotating is the inverted one.
(Reward – a Nobel Prize!!!)

Other Non-Holonomic Dynamical Objects



Bronze Rattleback (top), "Euler's Disk" (bottom)

Theory of Rising COM of Tops

The theoretical study of the rise of the COM of spinning tops began at least as early as 1844.

There is still no complete theory of the motion.

The modern mathematical approach has moved from physical "top-ology" to topology.

Only experimental studies adequately probe the parameter space for the behavior of tops under the forces of gravity and friction.

Experiments with Various Objects

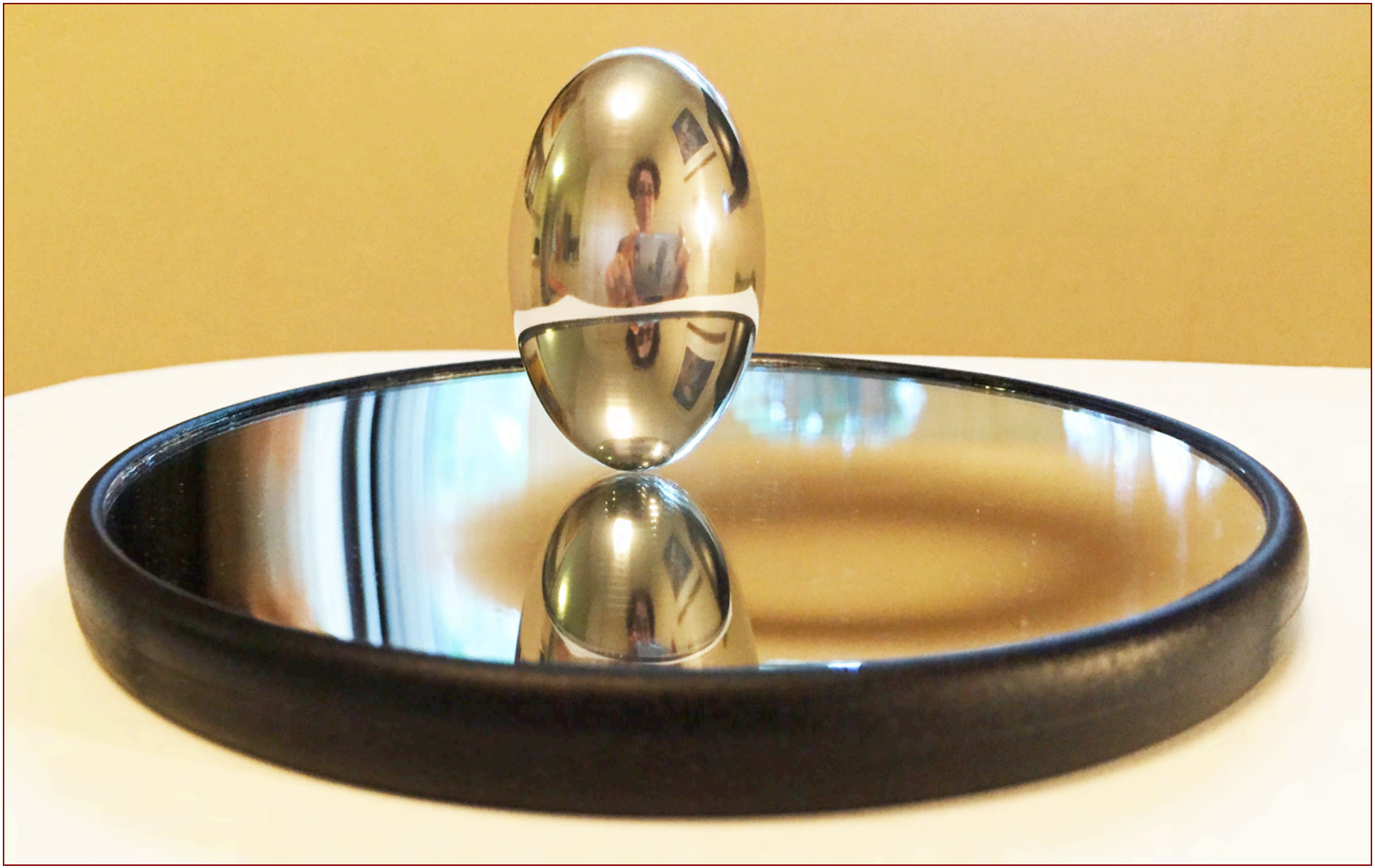
From left to right: (a) natural pebble; (b) polished agate stone egg; (c) classic Lingam stone; (d) ruby/fuschite Lingam stone; (e) black Lingam stone. Each is about 5 cm tall.

KB experiments with a range of objects indicate that only a finite range of prolate ellipsoidal shaped objects will raise their COM when spun on a horizontal surface and stand erect ($2 > c/a > 1$, $a = b$) where

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$$

Introducing the ΦTOP

Experimental studies of the rise of the COM using natural rocks, and a wide array of prolate ellipsoids made with rapid prototyping showed that objects with c/a in the range ~1.5 – 1.7 had the most appealing dynamics. Choosing c/a = Φ ~ 1.618.. one has the ΦTOP:



Why Φ?

Φ or Phi is the solution to the equation:

$1/\Phi = \Phi - 1$, so $\Phi \sim 1.618..$, $1/\Phi \sim .618..$

The golden mean, section, ratio or cut has been purported to have been built into works of art and architecture; to be found in natural biological objects such as shells and flowers; and is supposed to be preferred by humans when, for example, dividing a line into 2 parts.

PhiTOP in Action

Spun rapidly in a horizontal plane, it quickly stands erect, and can continue to spin in a vertical orientation for up to 3 minutes.

Some ΦTOP Properties

The PhiTOP includes the following properties:

Spun rapidly enough, it rises and stands erect

Spun slowly, it provides a novel visual illusion

Magnets can stop its spin, or spin it up

It has a visual and tactile aesthetic appeal

The ΦTOP and Its Cousins

(L) P. Hein "Super-egg", fiberglass, ~ 4 m tall, 1999
(C) R. Rhine and KB, "Golden Ellipsoid", wood, ~ 13 cm tall, 2015
(R) Gord Smith,"Superall", brass, ~18 cm tall, 1982

ΦTOP Summary

A new type of "philosophical toy" has been "devised" (or "designed" or "discovered", in any case optimized for its behavior and fabricated in various materials) which has novel dynamical, mathematical, visual and aesthetic properties.

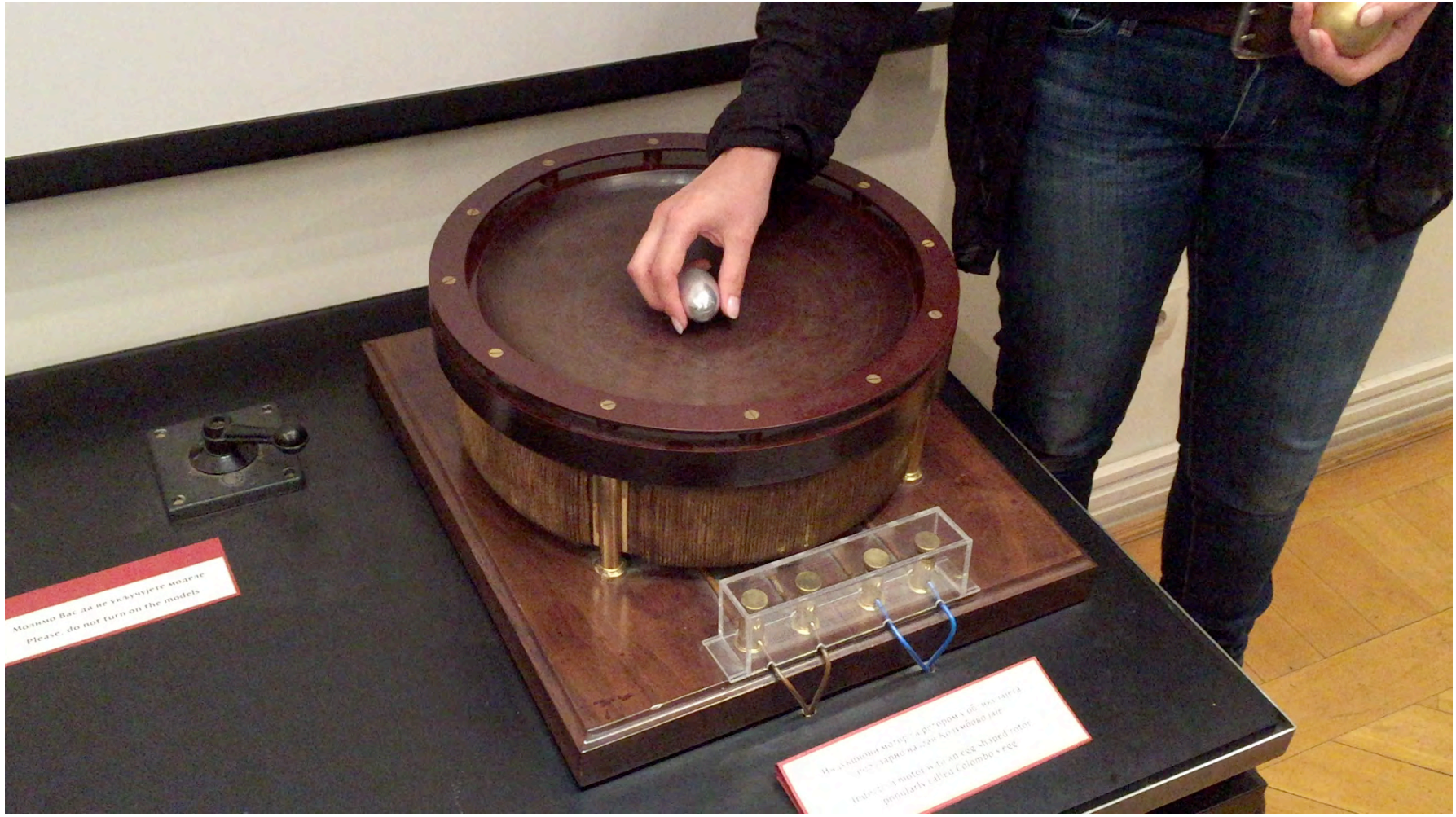
As soon as they arrive, rush out to a museum or store near you, get one and play with it!

Postscript
Teslas's "Egg of Columbus" Demonstration Using a ΦTOP

Kenneth Brecher
Departments of Astronomy and Physics
Boston University
Boston, MA 02215
E-mail: brecher@bu.edu
Telephone: 617-353-3423

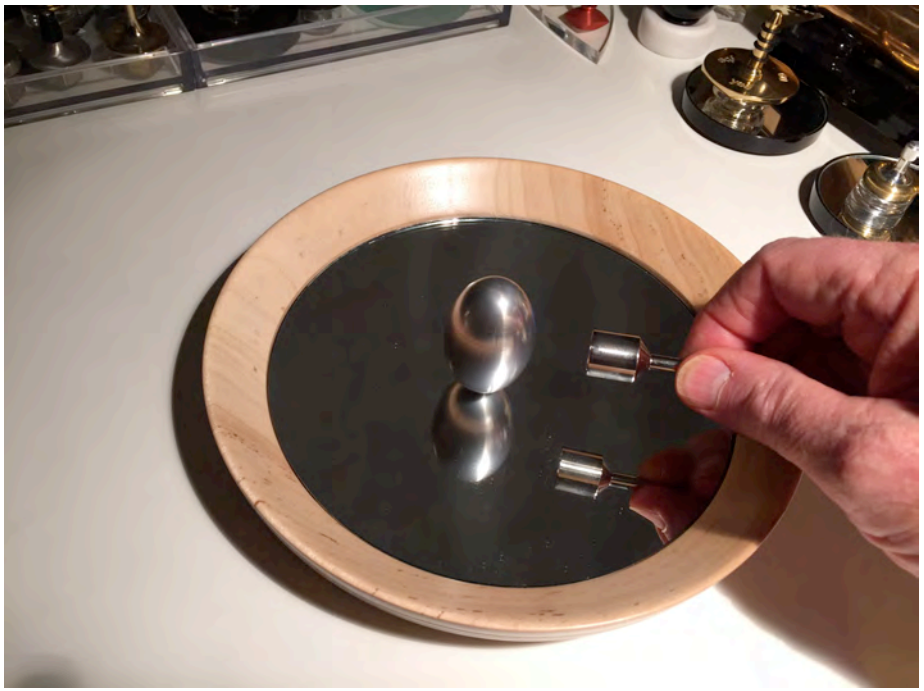
Hogarth's "Egg of Columbus"

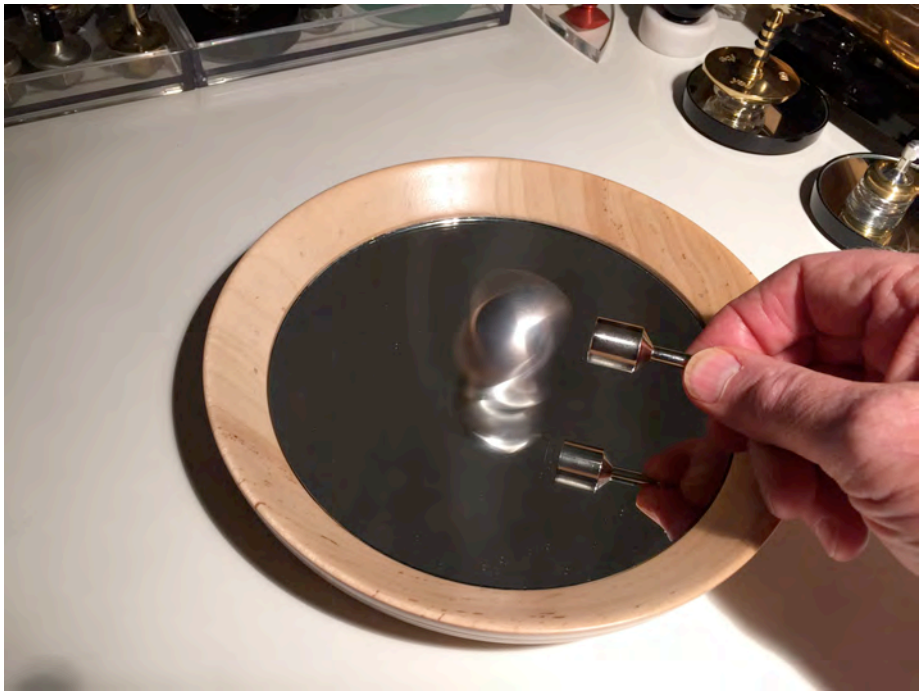
Tesla's "Egg of Columbus" Demonstration with ΦTOP



Spinning ΦTOP With Magnet

Tesla's "Egg of Columbus" demonstration uses AC power to create a rotating magnetic field by using several windings which can then spin up a conducting (but non-magnetic) object such as a copper egg. A static magnet held next to a spinning ΦTOP has the opposite effect.





Left: Spinning Aluminum ΦTOP Right: Spin Damped by Magnet