Top-ology:
A Torque* About Tops

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***(Pronounced "talk" as in "Yawk" when spoken with a New York accent)**

Why A Presentation About Tops?

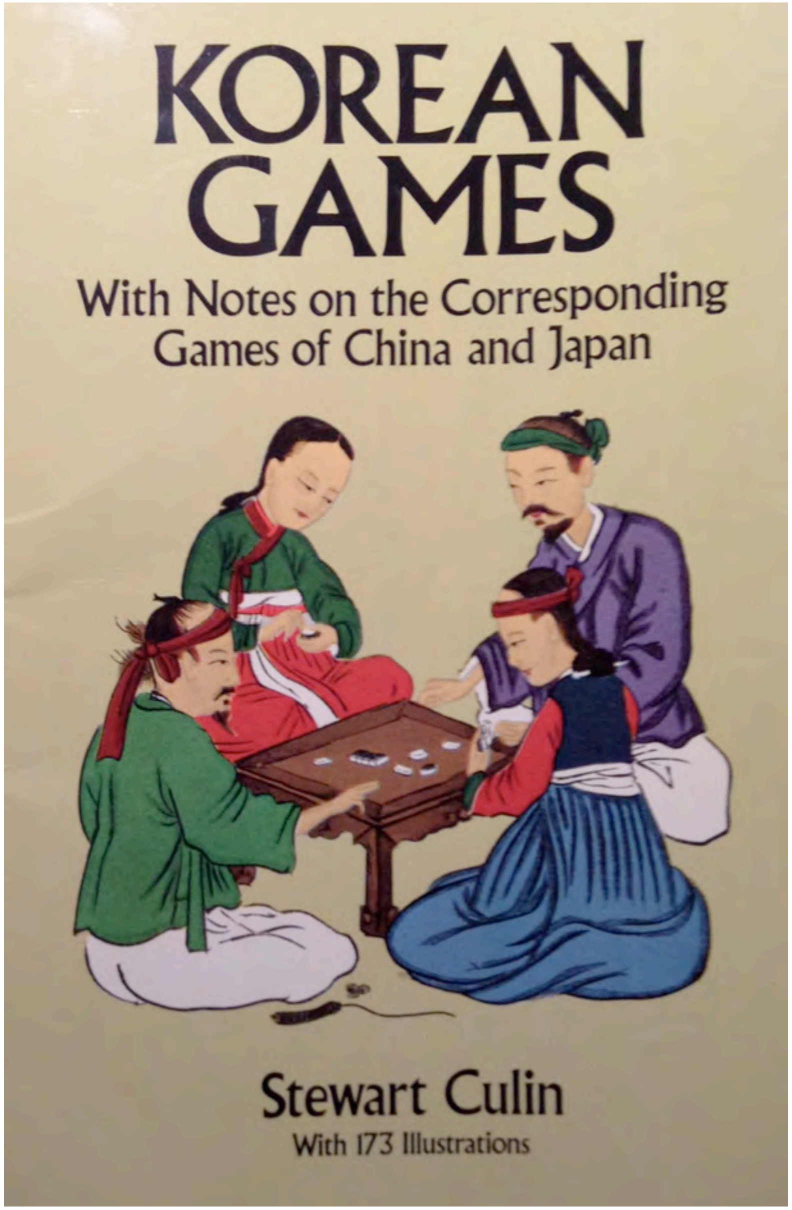
1. Share my enthusiasm for tops

2. Spark interest in the Bridges community

Hopefully leading to: novel top designs; new top paintings, graphics, sculptures, poetry, music, dance, literature, film, software, culture……
maybe even lead to mathematical insights

3. To premier "Top-ology: The Film," a new short movie made by Kaz and Ken Brecher

Korean Tops (팽이)



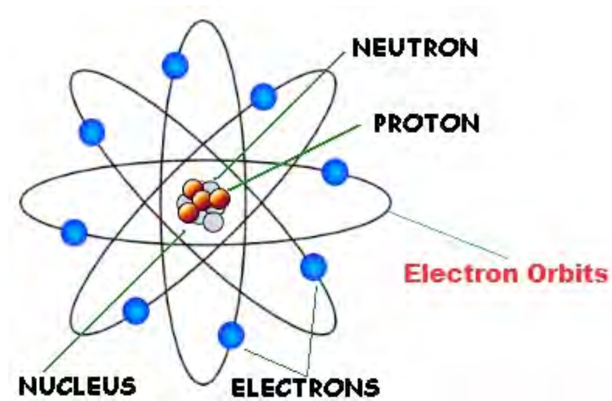
Debuted as an Olympic Sport in Korea in 1988:

Q: What Is A Top 

A: Physicists

Anything that can or does spin.

Since all matter is made of particles that spin – electrons, protons, neutrons, quarks – then everything is a top. (Except the Higgs Boson).



Photons (light) and gravitons (the carrier of gravity) also have spin (angular momentum)

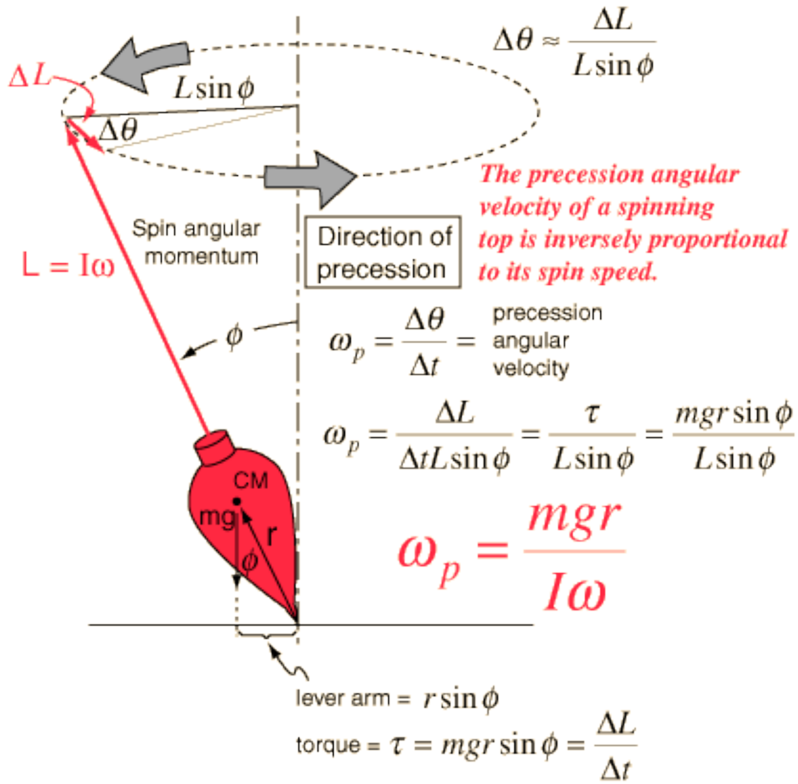
A: Astronomers

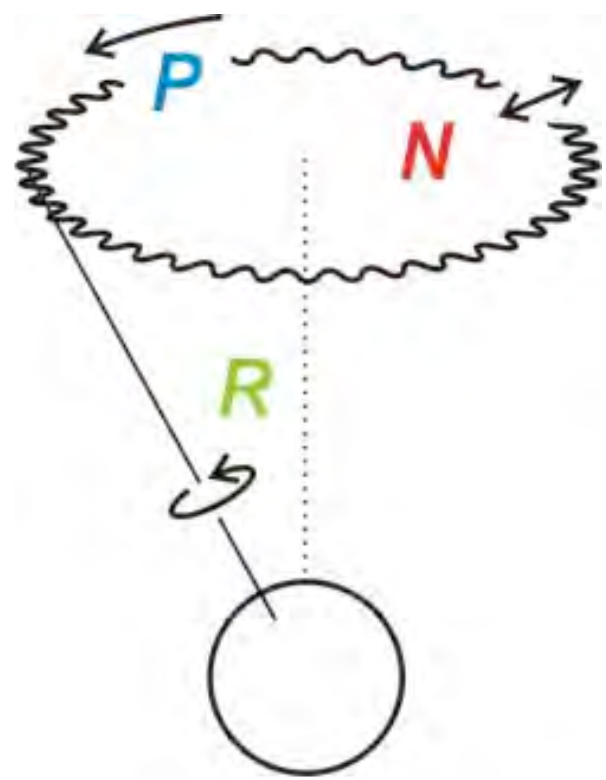
Many astronomical objects spin

Planets **Stars** **Galaxies**

A: Mathematicians

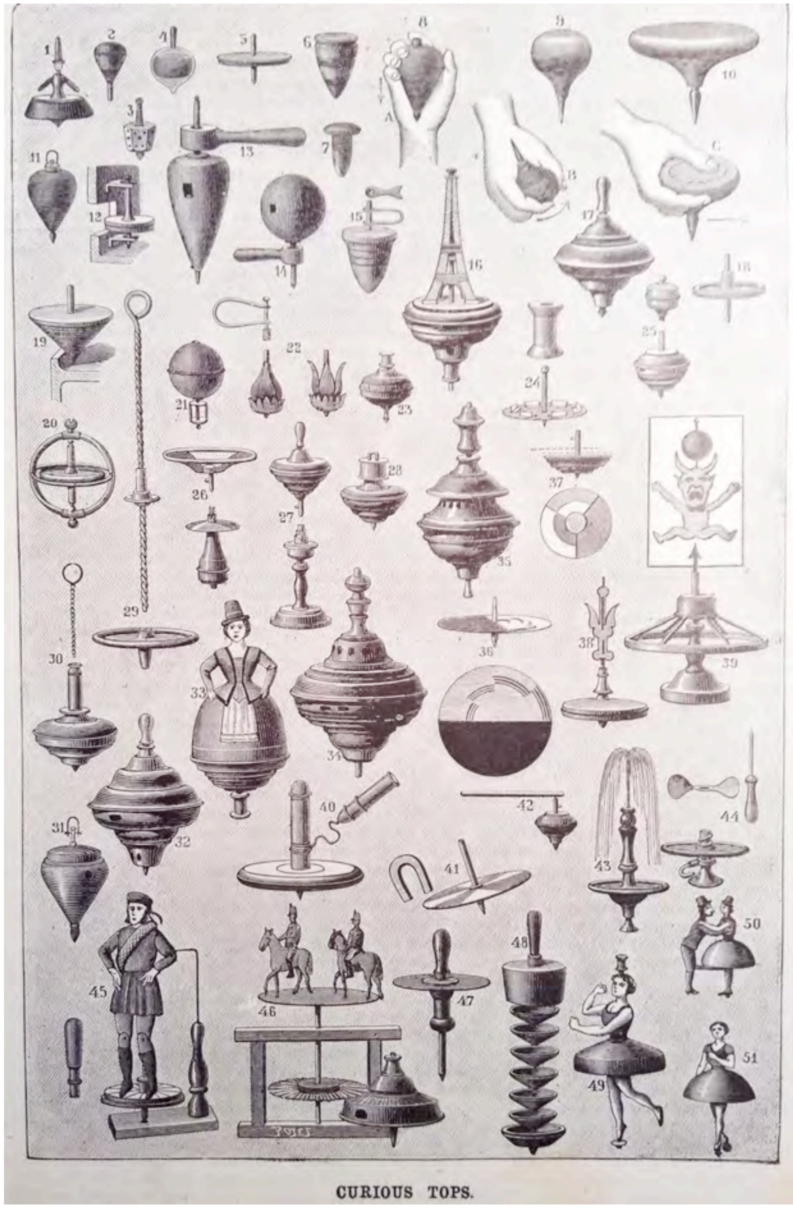
Only three closed form cases: (1) Euler Top - Spin (rotation) plus free precession (Earth with 24 hour spin plus Chandler Wobble);
(2) Lagrange Top: symmetric top moving about a fixed point with center of gravity on the symmetry axis (spin, precession & nutation);
(3) Kovaleskaya Top – special case. All the rest (with friction)????





A: Curious Kids & Cats

A: Topaholics



Origin of Spinning Tops

“The history of the Midianites is lost in the darkness of the ages and is not known; nonetheless scientists distinguish three distinctly separate periods in it: the first, about which nothing is known; the second about which one can say the same; and the third which followed the first two.” – A. T. Averchenko

The origin of spinning tops is also unknown.

Theories: Seed Pods? Sea Shells? Stones (celts)?

Brief History of Tops

Top from the tomb of Tutankhamen ca.1300 BC

Hittite bas-relief with whipping tops ca. 800 BC

Image on Greek vase of spinning top ca. 500 BC

Image on a Greek vase of a Yo-yo ca. 500 BC

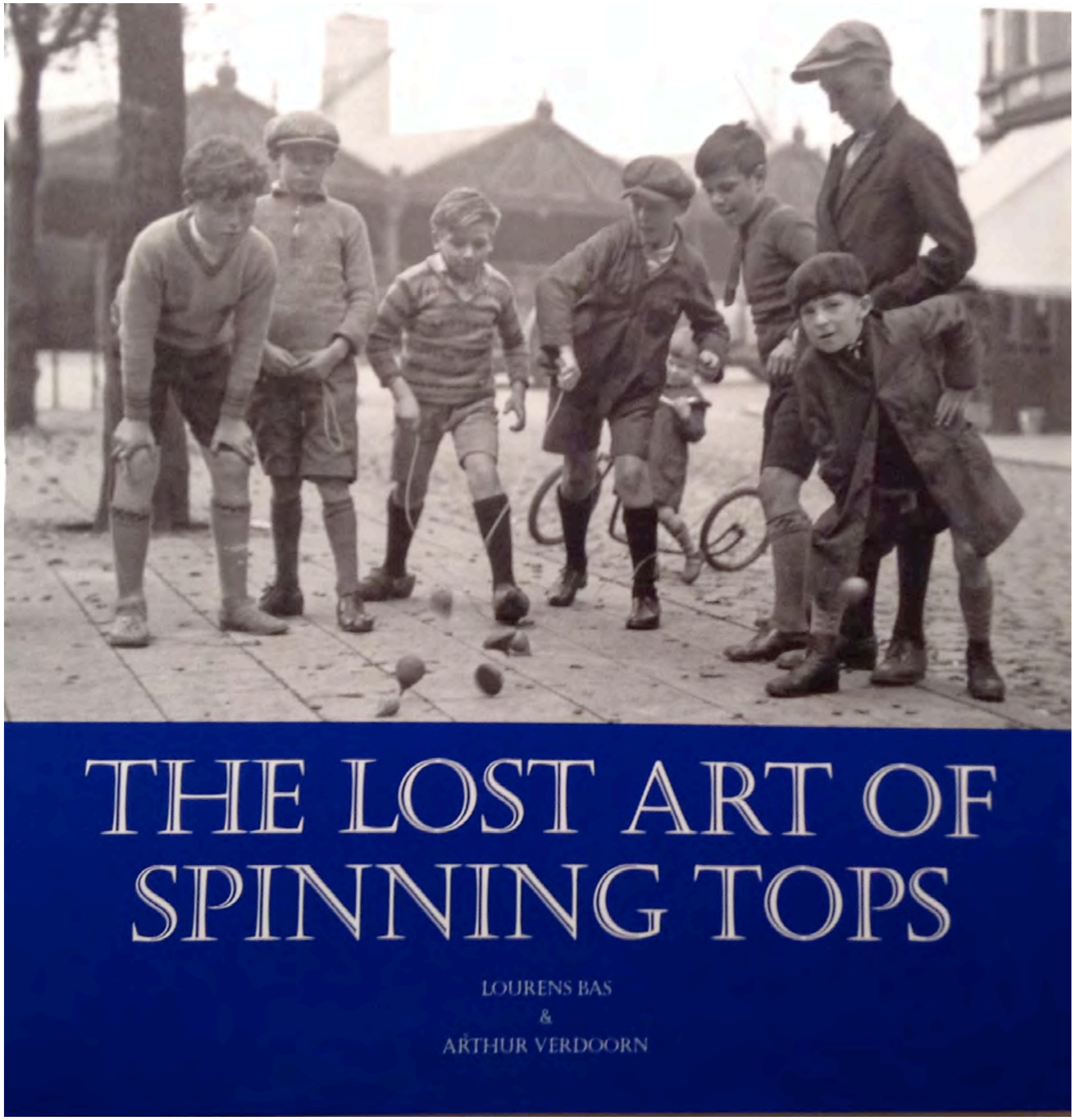
Greek Yo-yo ca. 460 BC

Mosaic of hoop game ca 600 AD, Constantinople

“Children’s Games” by Pieter Brueghel 1560

Detail from Brueghel's "Children's Games"

Peak Popularity in late 19th Century



And Today?

Gasing Tops in Kelantan, Malaysia, 2014

Olympic Ultimate Frisbee Next?

2500 years

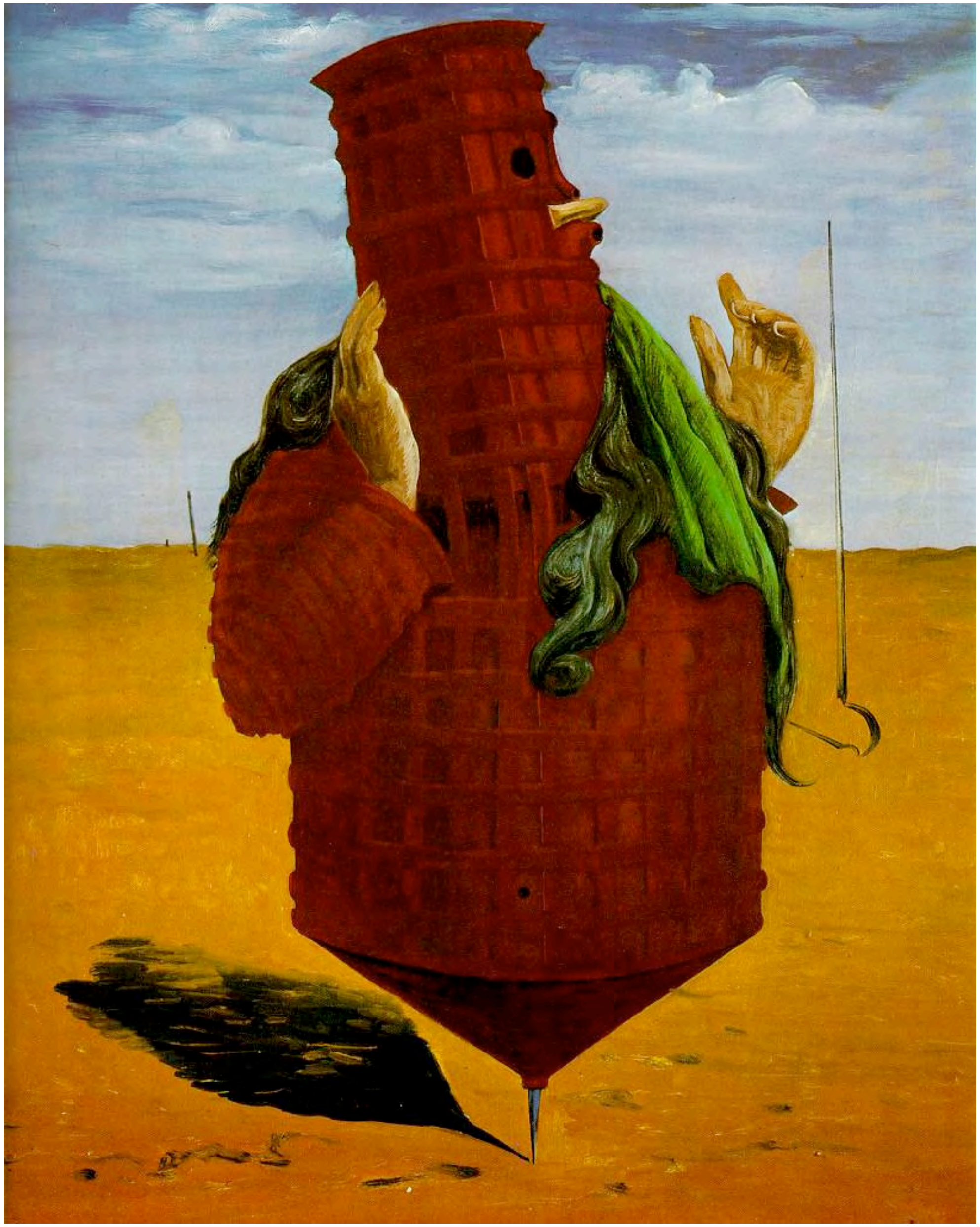
Tops in Sculpture

"Spinning Top" by Maori Sculptor Robert Jahnke (2002)

World’s largest (?) spinning top sculpture - in the Netherlands, ~ 10 m high, by Dutch artist Peter Hohmann

Tops in Paintings

"Boy with Spinning Top" (1750) Jean B. P. Chardin (Louvre, Paris)



“Ubu Imperator” (1923) by Max Ernst (Pompidou Center, Paris)

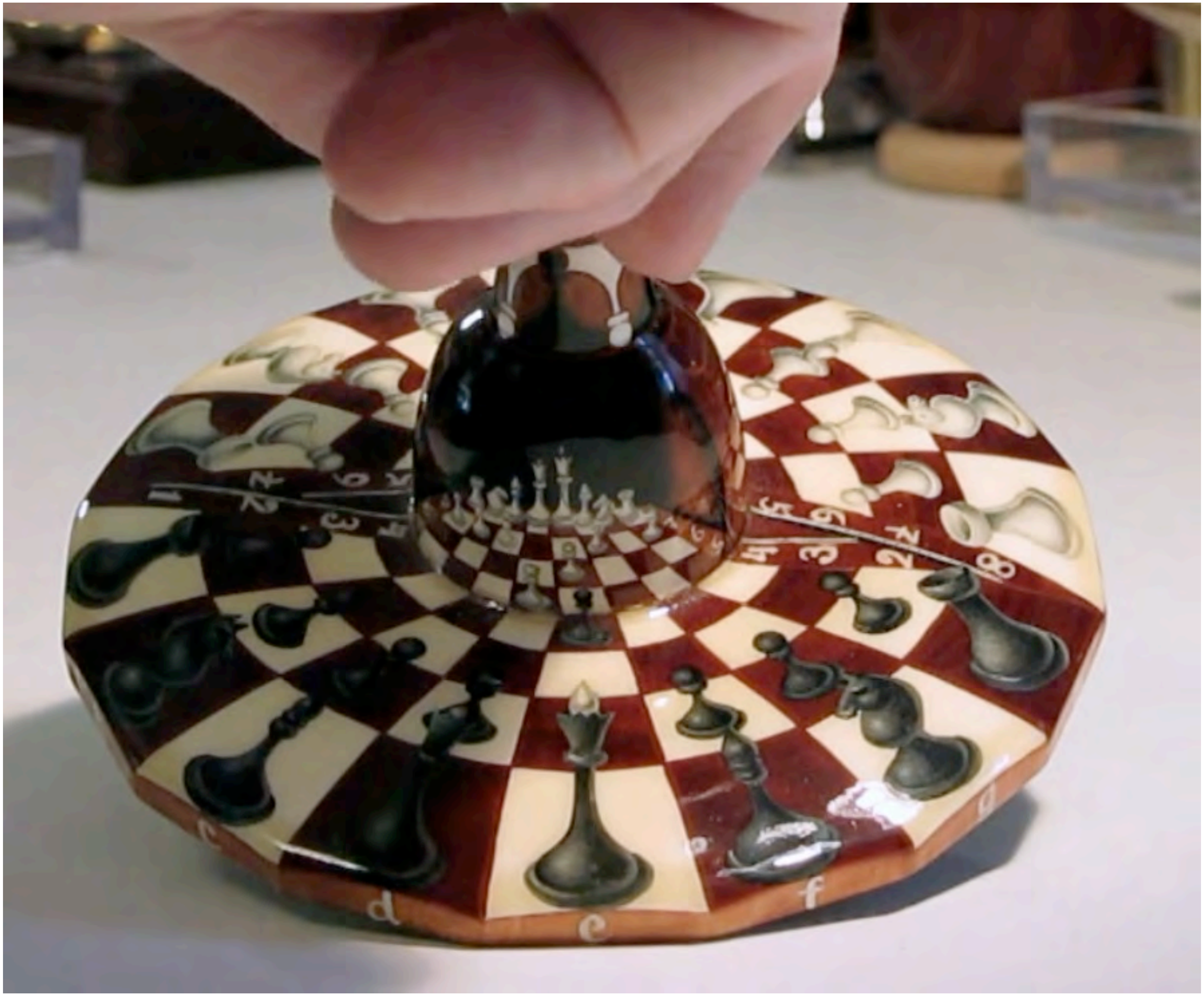
Tops as Art (or?)

Marcel Duchamp Bicycle Wheel Readymade (1913) MOMA

Marcel Duchamp's "Rotoreliefs" (1935) (Guggenheim NY)

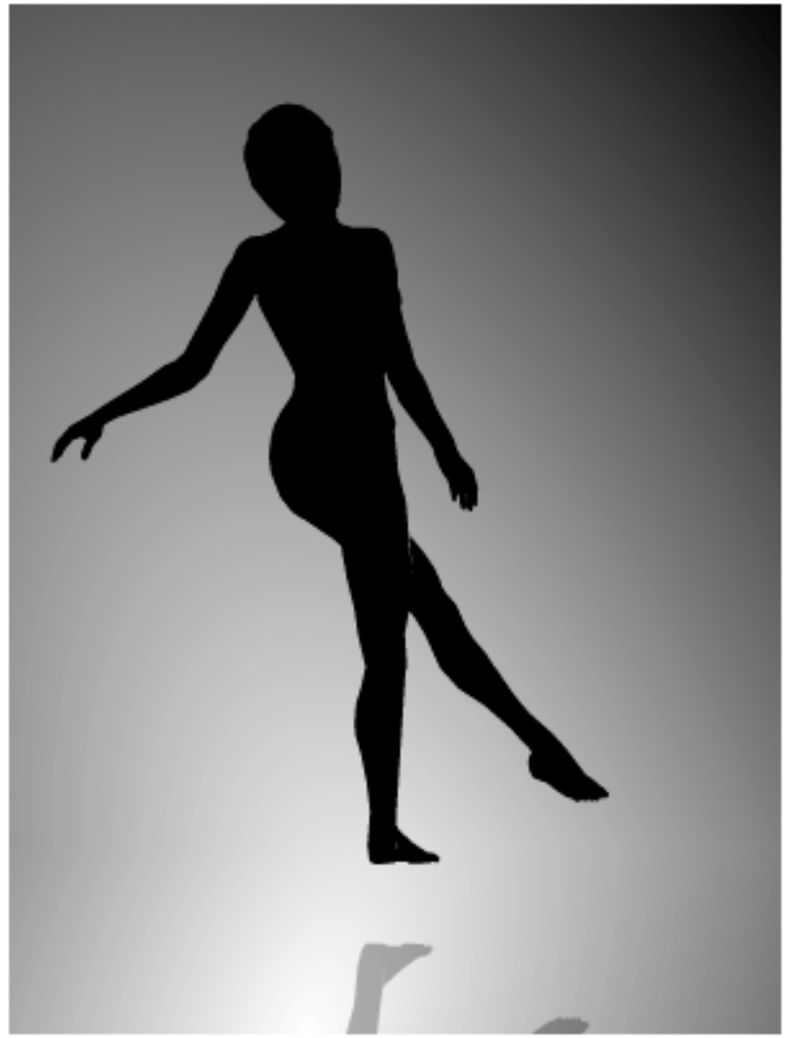
Tops as Craft (or Art?)

Tops by Randy Rhine (l.) & Armin Kolb (r.)



Chess Top by A. Bikyashev and I. Bobkova

Tops in Dance

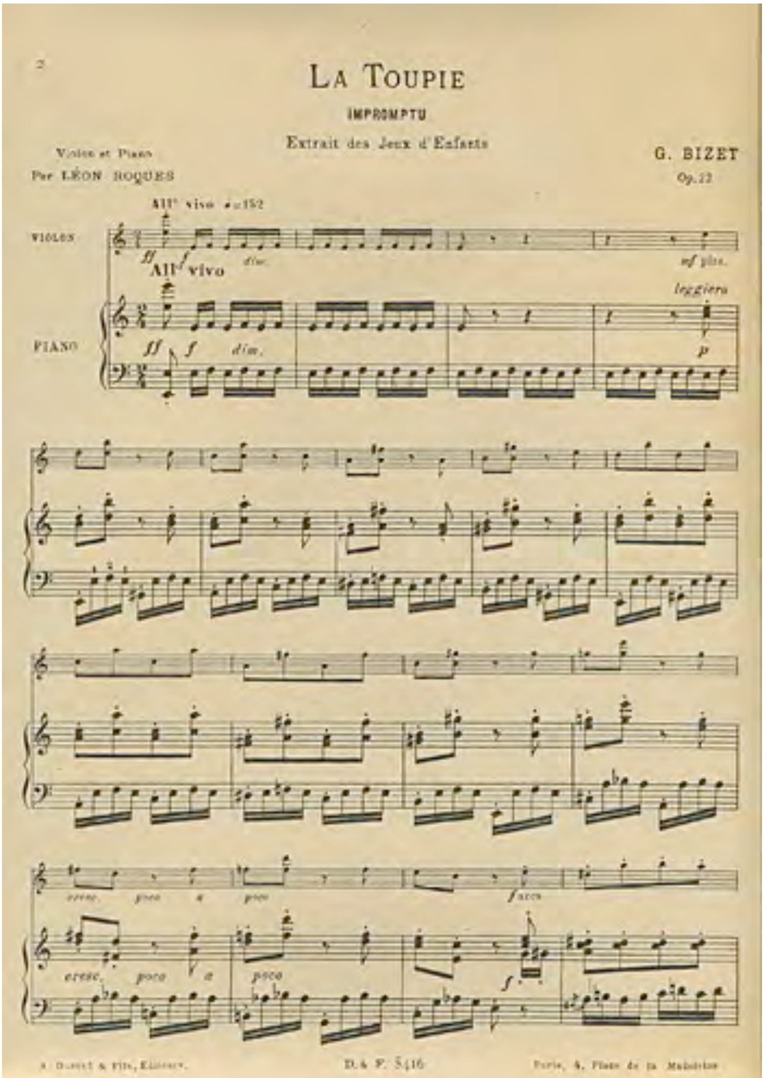


Ballet (real & ambiguous)

Whirling Dervishes

Nick Cave “Soundsuit” Costume/Sculpture

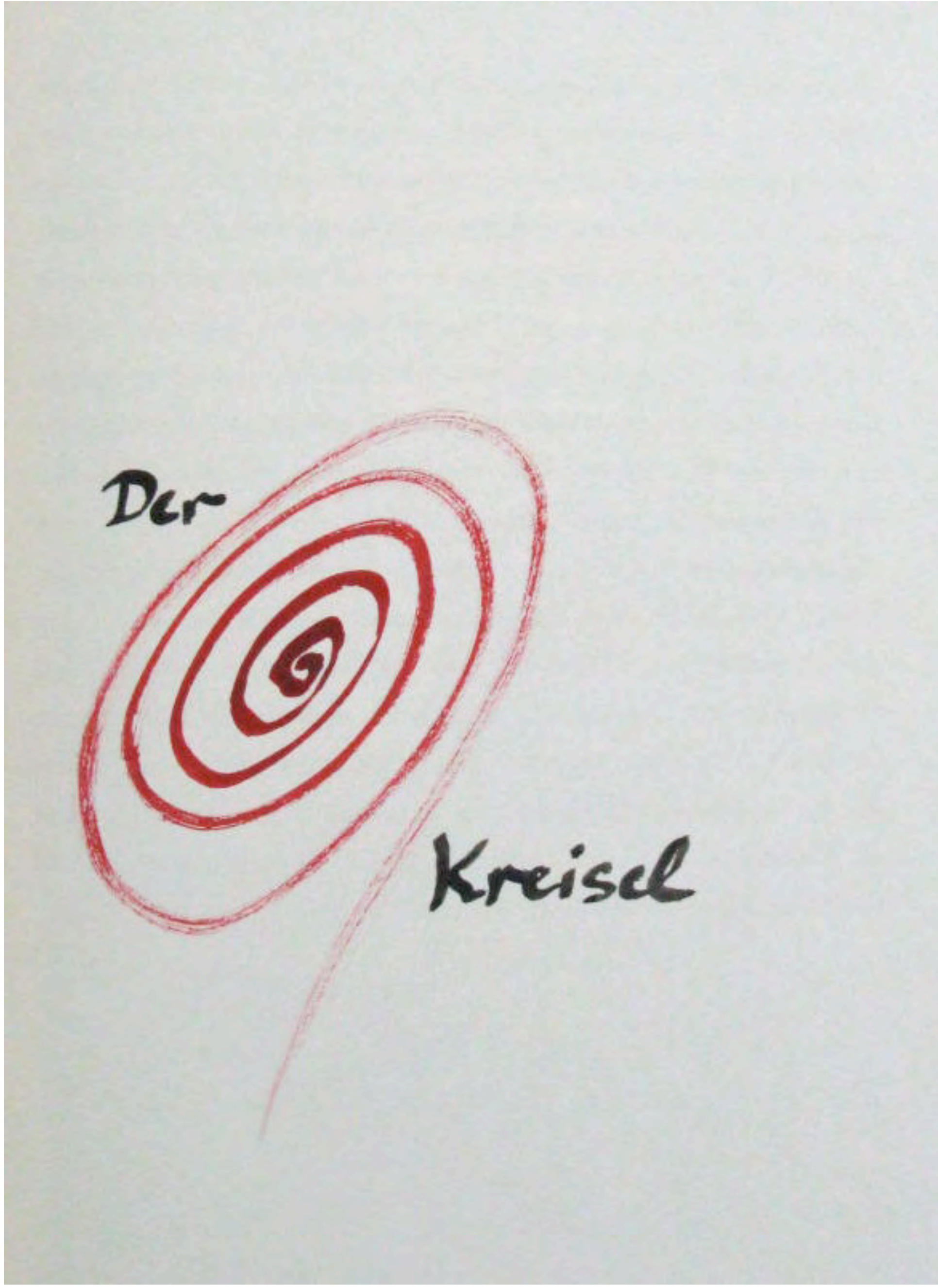
Tops in Music

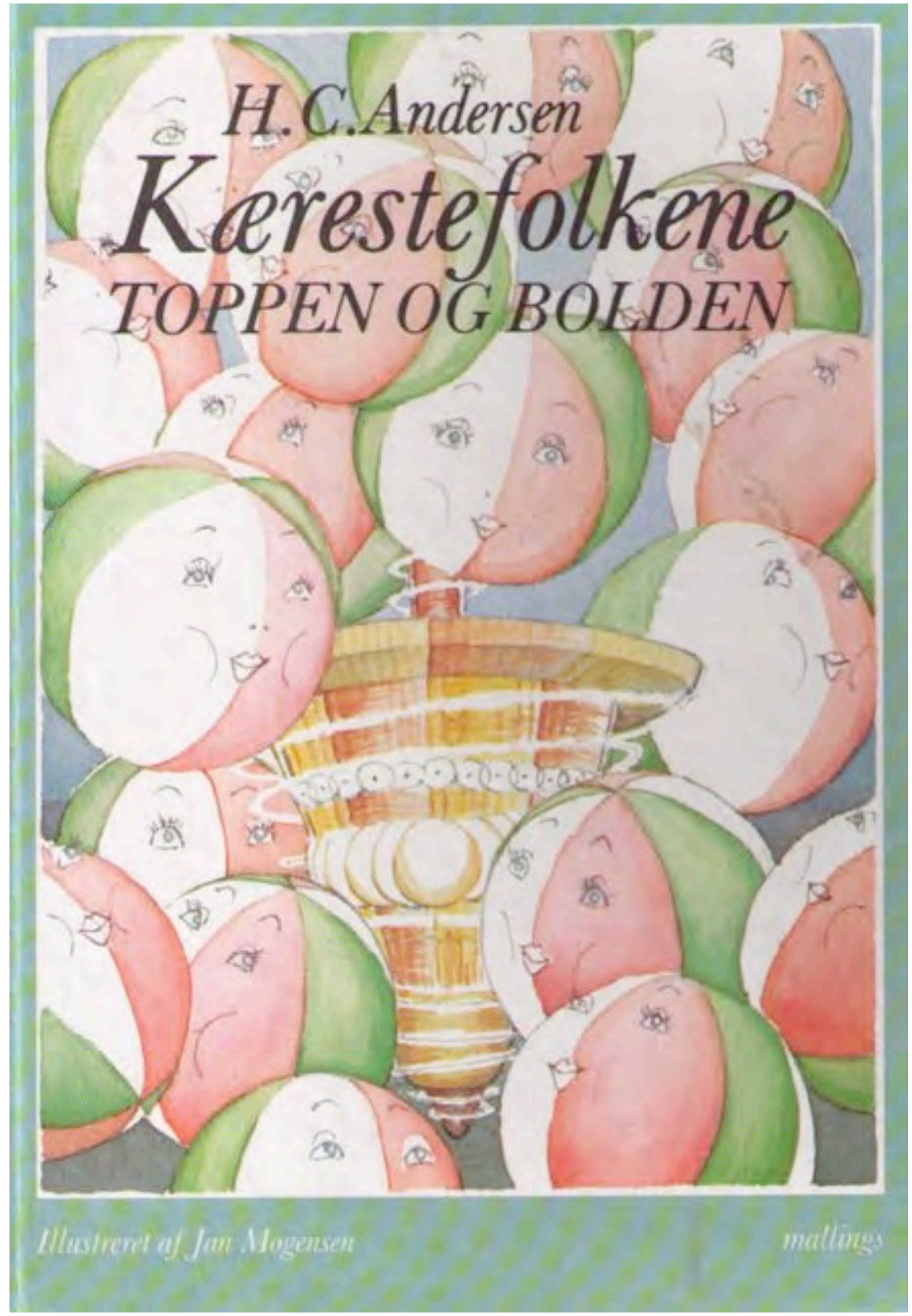


George Bizet's Impromptu "La Toupie" for two pianos

Tops in Literature

At least as early as Homer in the *Iliad* in 800 BC (writing about the fall of Troy which he said "…reels like a top staggering to its last turnings") tops are mentioned by writers, philosophers, scientists and poets throughout the past 3000 years. Among the references to tops by Shakespeare is this one in Coriolanus "…turned me about with his fingers and thumb, as one would set up a top." Short stories, and even entire novels, also feature spinning tops.





F. Kafka (l.) "Der Kreisel" and H. C. Anderson (r.)

Tops in Film

Film "tops" by Charles and Ray Eames (1969); stamp 2008

“Inception,” the film directed by C. Nolan uses a top (totem) for a reality check (2010)

Spinning Top Museums

Spinning Top Museums in Japan (l.) & U.S. (r.)

Tops in Mind

Isaac Newton: “These rays are not colored.”

James Clerk Maxwell: color top. Devised when he was 18, published 1856, started colorimetry.

Maxwell’s dynamical top ca. 1856

Project LITE Benham Color Top and Mach Band Top
PDFs of these and other patterns ready for printing and attaching to CDs are available at:

http://lite.bu.edu

Applied Top(ics)

Tops have been incorporated into gyroscopes, gyrostats, gyrocompasses, flywheels for ship stabilizers, governors and many other devices.

Sperry Gyroscope (l.), Centrifugal Governor (c.), Anschutz Gyrocompass (r.)

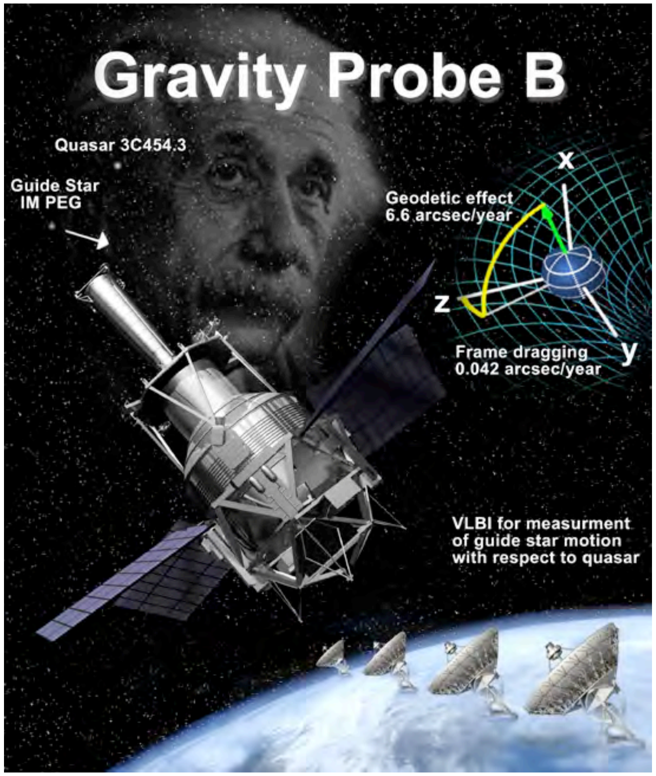
Invention of the Gyroscope

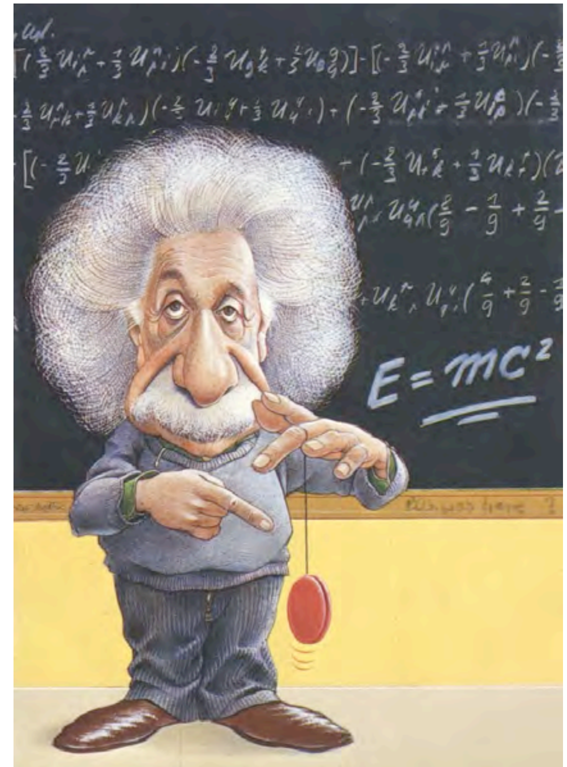


The gyroscope (which he called "the machine") was invented by German physicist (l.) J. G. F. Bohnenberger (University of Tubingen) ca. 1812 (first paper 1817). The example in the center is ca. 1813. The one on the right is ca.1850. It was well known in France & Germany by 1852, before the date of its supposed invention by Leon Foucault (who did give it its name and did use it show that the Earth spins on its axis).

New Bohnenberger Machines

Randy Rhine (with KB) 2012





Gyroscopes were used to test Einstein's General Theory of Relativity. The spinning quartz ball is about the size of a golf ball and is round to a part in a million (40 atoms). The Gravity Probe B experiment took 50 years to complete at a cost of ~ $750,000,000. Result: Einstein right again!

Counter Intuitive Tops

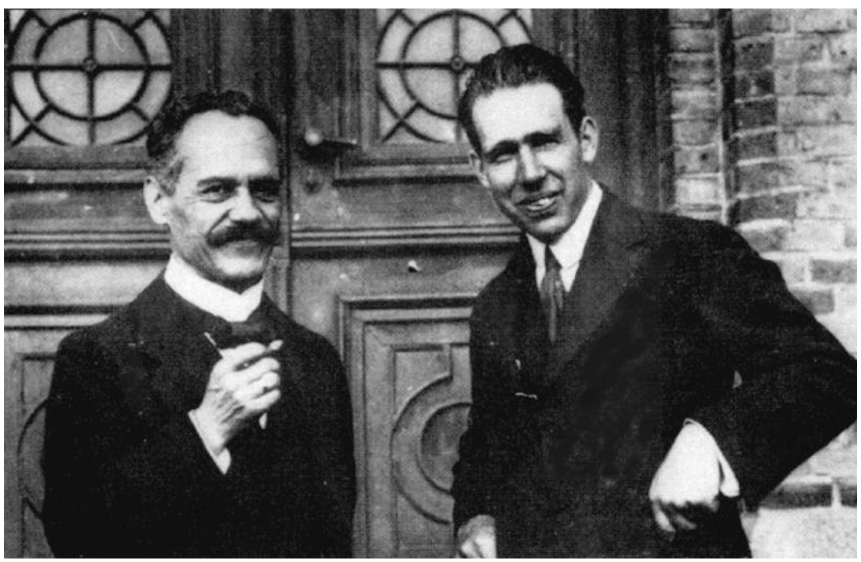
Physical systems acted upon by forces that only depend on position are called "holonomic."

Physical systems that are acted upon by forces such as friction, which depend on the speed of the motion, are called "non-holonomic."

Only the simplest holonomic tops have been quantitatively analyzed, even in the 4-volume, 1000 page treatise by F. Klein and A. Sommerfeld. Current mathematical treatments turn "top-ology" into topology.

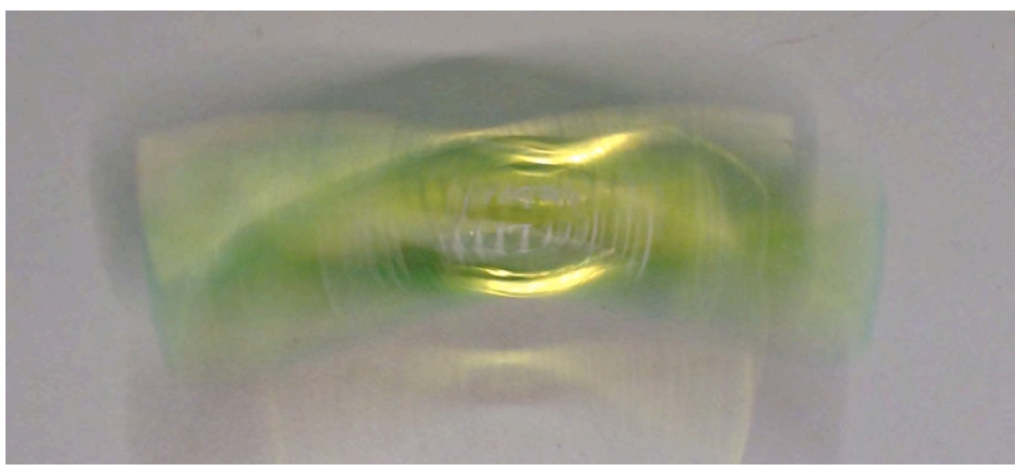
The Tippe-Top

spin & inverts to



A. Sommerfeld & N. Bohr (l.), W. Pauli & N. Bohr (r.)

Rattlebacks (Celts)




The Toycrafter
Have fun!
Project LITE
AeroSpinner

Designing Tops

Accreting Black Hole Top
(Design – KB; fabrication - R. Bischoff), ~22 cm diameter

Large "supported" top (KB & J. McClure), ~ 33 cm diameter.

Brass "finger" top (design KB, fabrication BU machine shop), ~ 6 cm diameter. What is the maximum time a finger top can spin?

“Discovering” Tops

Neodymium magnet balls top (KB)
(Design or discovery?)

Tom Dixon (designer) door stop/paperweight to Jack(-s) top by adding rubber "o-rings" (KB)

Cosmetics Jar lid to “osculating” top by inversion (KB)

Atomium (Brussels) (l.) and “Atomium top” by KB (r.)

Coins combine spinning and rolling to produce "spolling." They settle down with decreasing spin frequency but increasing rolling frequency. "Euler's disc" is a commercial version. It exhibits a finite time singularity.

Euler's disk and its finite-time singularity

Air viscosity makes the rolling speed of a disk go up as its energy goes down.

It is a fact of common experience that if a circular disk (for example, a penny) is spun upon a table, then ultimately it comes to rest quite abruptly, the final stage of motion being characterized by a shudder and a whirring sound of rapidly increasing frequency. As the disk rolls on its rim, the point P of rolling contact describes a circle with angular velocity Ω. In the classical (non-dissipative) theory[1], Ω is constant and the motion persists forever, in stark conflict with observation. Here I show that viscous dissipation in the thin layer of air between the disk and the table is sufficient to account for the observed abruptness of the settling process, during which, paradoxically, Ω increases without limit. I analyse the nature of this 'finite-time singularity', and show how it must be resolved.

Let α be the angle between the plane of the disk and the table. In the classical description, and with the notation defined in Fig. 1, the points P and O are instantaneously at rest in the disk, and the motion is therefore instantaneously one of rotation about line PO with angular velocity ω, say. The angular momentum of the disk is therefore $\mathbf{h} = A\omega\mathbf{e}(t)$, where $A = \frac{1}{4}Ma^2$ is the moment of inertia of the disk of mass M about its diameter; $\mathbf{e}(t)$ is a unit vector in the direction PO; $\mathbf{e}_z$, $\mathbf{e}_d$ are unit vectors in the directions Oz, OC, respectively (see Fig. 1). In a frame of reference rotating with angular velocity $\boldsymbol{\Omega}_d = \Omega\mathbf{e}_z$, the disk rotates about its axis OC with angular velocity $\boldsymbol{\Omega}_d = \Omega_d\mathbf{e}_d$; hence the rolling condition is $\Omega_d = \Omega\cos\alpha$. The absolute angular velocity of the disk is thus $\boldsymbol{\omega} = \Omega(\mathbf{e}_d\cos\alpha - \mathbf{e}_z)$, and so

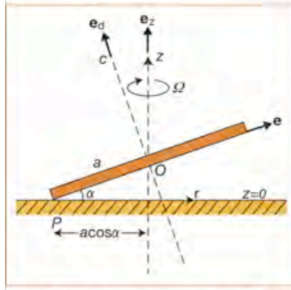


Figure 1 A heavy disk rolls on a horizontal table. The point of rolling contact P moves on a circle with angular velocity Ω. Owing to dissipative effects, the angle α decreases to zero within a finite time and Ω increases in proportion to $\alpha^{-1/2}$.

$\omega = \boldsymbol{\omega}\cdot\mathbf{e} = -\Omega\sin\alpha$.

Euler's equation for the motion of a rigid body is here given by $\mathrm{d}\mathbf{h}/\mathrm{d}t = \boldsymbol{\Omega}\wedge\mathbf{h} = \mathbf{G}$, where $\mathbf{G} = Mga\mathbf{e}_z\wedge\mathbf{e}$ is the gravitational torque relative to P ($\wedge$ indicates the vector product). This immediately gives the result $\Omega^2\sin\alpha = 4g/a$, or, when α is small,

$$\Omega^2\alpha \approx 4g/a \qquad (1)$$

The energy of the motion E is the sum of the kinetic energy $\frac{1}{2}A\omega^2 = \frac{1}{2}Mga\sin\alpha$, and the potential energy $Mga\sin\alpha$, so

$$E = \tfrac{3}{2}Mga\sin\alpha \approx \tfrac{3}{2}Mga\alpha \qquad (2)$$

In the classical theory, α, Ω and E are all constant, and the motion continues indefinitely. As observed above, this is utterly unrealistic.

Let us then consider one of the obvious mechanisms of energy dissipation, namely that associated with the viscosity μ of the surrounding air. When α is small, the dominant contribution to the viscous dissipation comes from the layer of air between the disk and the table, which is subjected to strong shear when Ω is large. We may estimate the rate of dissipation of energy in this layer as follows. Let (r,θ) be polar coordinates with origin at O. For small α, the gap $h(r,\theta,t)$ between the disk and the table is given by $h(r,\theta,t) \approx \alpha(a + r\cos\phi)$, where $\phi = \theta - \Omega t$. We now concede that α is a slowly varying function of time t: we assume that $|\dot\alpha| \ll \Omega$, and make the 'adiabatic' assumption that equation (1) continues to hold. Because the air moves a distance of order a in a time $2\pi/\Omega$, the horizontal velocity $\mathbf{u}_H$ in the layer has order of magnitude $r\Omega\sin\phi$; and as this velocity satisfies the no-slip condition on $z = 0$ and on $z = h$ $(= O(\alpha a))$, the vertical shear $|\partial\mathbf{u}_H/\partial z|$ is of the order $(r\Omega/\alpha a)|\sin\phi|$. The rate of viscous dissipation of energy Φ is given by integrating $\mu(\partial\mathbf{u}_H/\partial z)^2$ over the volume V of the layer of air: this easily gives $\Phi \approx \pi\mu g a^2/\alpha^2$, using equation (1). The fact that $\Phi \to \infty$ as $\alpha \to 0$ should be noted.

The energy E now satisfies $\mathrm{d}E/\mathrm{d}t = -\Phi$ (neglecting all other dissipation mechanisms). Hence, with E given by equation (2), it follows that

$$\tfrac{3}{2}Mga\,\mathrm{d}\alpha/\mathrm{d}t = -\pi\mu g a^2/\alpha^2 \qquad (3)$$

This integrates to give

$$\alpha^3 = 2\pi(t_0 - t)/t_1 \qquad (4)$$

where $t_1 = M/\mu a$, and t_0 is a constant of integration determined by the initial condition: if $\alpha = \alpha_0$ when $t = 0$, then $t_0 = (\alpha_0^3/2\pi)t_1$. What is striking here is that, according to equation (4), α does indeed go to zero at the finite time $t = t_0$. The corresponding behaviour of Ω is $\Omega \approx (t_0 - t)^{-1/6}$, which is certainly singular as $t \to t_0$.

Of course, such a singularity cannot be realized in practice: nature abhors a singularity, and some physical effect must intervene to prevent its occurrence. Here it is not difficult to identify this effect: the vertical acceleration $|\ddot h| = |a\ddot\alpha|$ cannot exceed g in magnitude (as the normal reaction at P must remain positive). From equation (4), this implies that the above theory breaks down at a time τ before t_0, where

$$\tau = t_0 - t \approx (2a/9g)^{3/5}(2\pi/t_1)^{1/5} \qquad (5)$$

A toy, appropriately called Euler's disk[2], is commercially available (Fig. 2; Tangent Toys, Sausalito, California). For this disk, $M = 400$ g, and $a = 3.75$ cm. With these values and with $\mu = 1.78 \times 10^{-4}$ g cm^{-1} s, $t_1 = M/\mu a \approx 0.8 \times 10^6$ s, and, if we take $\alpha_0 = 0.1 (\approx 6°)$, we find $t_0 \approx 100$ s. This is indeed the order of magnitude (to within $\pm 20\%$) of the observed settling time in many repetitions of the spinning of the disk (with quite variable and ill-controlled initial conditions), that is, there is no doubt that dissipation associated with air friction is sufficient to account for the observed behaviour. The value of τ given by equation (5) is 10^{-2} s for the disk values given above; that is, the behaviour described by equation (4) persists until within 10^{-2} s of the singularity time t_0. At this stage, $\alpha \approx 0.5 \times 10^{-2}$, $h_0 = a\alpha \approx 0.2$ mm, $\Omega \approx 500$ Hz (and the adiabatic approximation is still well

Figure 2 Euler's disk is a chrome-plated steel disk with one edge machined to a smooth radius. If it were not for friction and vibration, the disk would spin for ever. Photo courtesy of Tangent Toys. See http://www.tangenttoy.com/.

Tibetan humming (singing) bowl - designed for acoustics. Some of the bowls have a curved bottom. They rise up when spun around their symmetry axis - the opposite of coins. Then they spin and wobble: "spobble." (KB discovery)

Summary

Anything can be or is a top

Most tops are engaging

Many tops remain puzzling

So start spinning things!!!!

Acknowledgments

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Toupie Or Not To Be