The Sirius Enigmas Mathematical Tops

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Abstract

I have developed a set of four spinning tops based on four of the most important mathematical constants [1]: ϕ, π, e and i. The tops are quite elegant, have different topological shapes and have unusual dynamical properties. Here, I discuss each of them separately, as well as a mathematical relation that unites them together.

PhiTOP® Development

In 2015, I did an experimental study of the problem of the rise of the center of mass of spinning objects, focusing on ovoids (eggs) and prolate ellipsoids. I presented the first results of this study at a meeting of the American Physical Society in April 2015. By the Spring of 2015, I had found experimentally that the "optimal" prolate ellipsoidal shaped object (optimal in the sense that it could be spun up easily, rise quickly and stand erect for a long time) has a ratio of major to minor axes of about 1.6. I decided to have such objects fabricated in various materials, chose the major to minor axis ratio to be equal to the "golden mean" ("golden ratio") $\phi \sim 1.618\ldots$, and named the resulting object the "PhiTOP®" or "ϕTOP®". The PhiTOP® was first presented in a paper entitled "The PhiTOP: A Golden Ellipsoid" at the Bridges conference on art and mathematics held in July 2015 [2]. More about the PhiTOP® can be found at: http://www.thephitop.com, in reference [3] as well as in the description in U.S. patent # 9,561,446.

PiTOP® Development

During 2016 and 2017, I studied the physics of spinning and rolling coins and flattened disks (right circular cylinders). As in the case of spinning egg-shaped objects, there is a lengthy literature of the physics of spinning coins that dates back at least to the 19th century. As with the PhiTOP®, I conducted a series of experiments to determine the "optimal" coin or disk (that is, one that spins and precesses for the longest time). The maximum duration spinning and rolling (precession) time was found to depend mainly on the ratio of the disk radius r to disk thickness t. I found experimentally that with $r/t \sim 3$ one produced the longest duration motion. I then chose r/t to be equal to $\pi \sim 3.1415\ldots$ and named the resulting object the "PiTOP®" or "πTOP®". Its volume V is exactly equal to r^3. The design that appears on one surface consists of a spiral containing the first 109 digits of π along with the Greek letter π in the middle. The PiTOP® was first presented at the "Gathering for Gardner" (G4G13) event in Spring 2018. More about the PiTOP® can be found at: http://www.thepitop.com.

Figure 1 (from left to right): *(a) brass ϕTOP®; (b) brass πTOP®; (c) brass iTOP; (d) brass eTOP.*

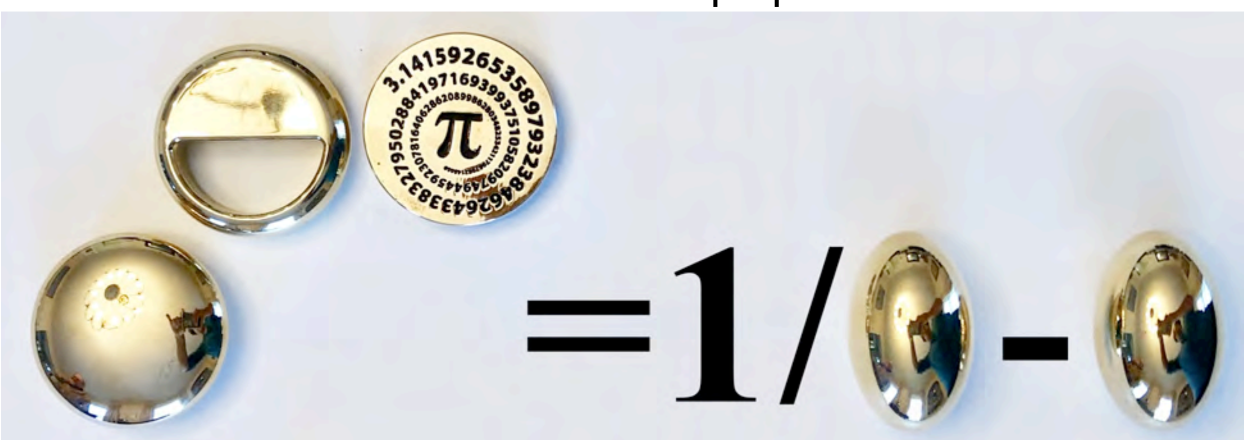
*e*TOP Background

Having designed both the PhiTOP® and the PiTOP®, I decided that the world also needed an "*e*TOP". In this case, no optimizing physics experiments were done. I simply chose to produce an oblate ellipsoid with the ratio of diameter d to maximum thickness t having $d/t = e \sim 2.718\ldots$ It can be spun like a coin. While spinning in its upright position, it presents a visible cross-section quite similar to that of the upright spinning PhiTOP®! As it settles down, it gives rise to beautiful Lissajous figure like reflections.

*i*TOP Background

Finally, I felt that the world also needed an imaginary top or "*i*TOP". After pondering the question of what such a thing might be and look like, I decided that a "real" imaginary top was an unlikely proposition (though check Nick Bantock's ideas about such things in his book "The History of Imaginary Spinning Tops" [4]). Therefore, I devised the "Inverting Ring Top", or "*i*TOP", a quasi two-dimensional "tippe-top" to join the other three "Sirius Enigmas" tops. By combining the definition of the golden mean $1/\phi - \phi = -1$ with Euler's equation: $e^{i\pi} = -1$ one has a relation connecting *e*, *i*, π and ϕ:

$$e^{i\pi} = 1/\phi - \phi.$$



Figures 2: A physical/mathematical relation between all of the Sirius Enigmas Spinning Tops.

Summary

In conclusion, I have devised a set of four unique spinning tops, each with its own novel physical and mathematical properties. Each has a longest dimension of 2 inches. When made in brass, each weighs between 4 and 8 ounces (though some of them have been produced in many other materials including aluminum, copper, titanium, bronze, stainless, steel, glass, plastic and wood). The four different types of "Sirius Enigmas" spinning tops can be thought of as being related through a mathematical identity.

Acknowledgments

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References

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