## The Sirius Enigmas Mathematical Tops

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#### Overview

The physics, mathematics and sheer beauty of spinning objects - tops, dreidels, gyroscopes, rattlebacks, planets, stars, black holes, galaxies - intrigues and delights everyone.

Having already introduced two new spinning tops to the world at previous G4G gatherings the  $\phi$ TOP<sup>®</sup> at G4G12 - and the  $\pi$ TOP<sup>®</sup> at G4G13 - today I will introduce two new mathematical tops: the *e*Top and the *i*TOP.

## Constants $\phi$ , $\pi$ , *e* and *i*



D. Perkins, " $\phi$ ,  $\pi$ , *e* and *i*", Mathematical Association of America, 2018.

## **The \phi TOP^{\mathbb{R}}** The PhiTOP<sup>®</sup> is a prolate ellipsoid with ratio of major to minor axes equal to $\phi \sim 1.618...$





Cf. "Physics of the PhiTOP", K. Brecher and R. Cross, <u>The Physics Teacher</u>, 57, 74, 2019.

#### The $\pi TOP^{\mathbb{R}}$

The PiTOP<sup>®</sup> is a right circular cylinder with ratio of radius r to thickness t,  $r/t = \pi \sim 3.14...$ 



# πτοp<sup>®</sup> Dynamics

Spin the PiTOP<sup>®</sup> as you would a coin. It will spin but also precess. As the PiTOP<sup>®</sup> loses kinetic energy to friction, the angle  $\alpha$  it makes with the surface will decrease and the precession frequency  $\Omega$  increases.



Data based on the dynamics of the PiTOP<sup>®</sup> from Rod Cross, "Effects of Rolling Friction on a Spinning Coin", <u>European Journal of Physics</u>, 39, #3, 2018.

#### **The** *e***TOP** The *e***TOP** is an oblate ellipsoid with ratio of major/minor axes = $e \sim 2.718...$



## eTOP Dynamics

The *e*TOP, like the  $\pi$ TOP, can be spun by hand starting from a standing position. It can also be spun up with a magnetic stirrer.



## **Imaginary Tops**





Nick Bantock is the author "The Museum at Purgatory", among many other books.

#### *i*TOP – Inverting Top A "real" imaginary top is, well, hard to imagine. In its place, I have designed a quasi two-dimensional inverting top – the "*i*TOP".



#### **iTOP Dynamics**

Spin the iTOP starting from a standing position, heavy side down, and it almost instantly inverts to spin in the opposite direction! Imagine that! Almost an imaginary top!



#### **The Sirius Enigmas Mathematical Tops**



## Euler's Identity $e^{i\pi} = -1$

**Combined with the definition of**  $\phi$ 

 $\frac{1}{\phi} - \phi = -1$ Results in the "*e*, *i*,  $\pi$ ,  $\phi$  " Identity:  $e^{i\pi} = 1/\phi - \phi$ 

## Sirius Enigmas Tops Physical Identity



#### **Postscript: G4G14 Cuboctahedron Top**







#### **Post Postscript** Introducing the DeltaCelt<sup>®</sup>

There is one more universal mathematical constant:  $\delta \sim 4.669.....$  the Feigenbaum constant. In 2019 I designed a new rattleback or celt based on  $\delta$ . Like the other Sirius Enigmas spinning tops, it is quite elegant. Its motion is counterintuitive. It is also adjustable.

